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Enzyme action optimizer: a novel bio-inspired optimization algorithm

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Abstract
This paper presents the enzyme action optimization (EAO) algorithm, a novel bio-inspired optimization algorithm designed to simulate the adaptive enzyme mechanism in biological systems. EAO employs a novel strategy that dynamically balances between exploration and exploitation to efficiently navigate and optimize complex, multi-dimensional search spaces. EAO has been tested over diverse benchmark datasets, including the 23 classical benchmark functions, IEEE CEC2017, CEC2022 benchmark functions, where it has been compared with 14 recent and highly cited optimizers. The results show the superior performance of EAO over the compared optimizers in terms of finding the optimal solution, convergence speed, robustness, and overall performance. Furthermore, EAO was applied to solve five engineering design problems and demonstrated excellent performance results. The source code of EAO is publicly available for both MATLAB at: https://www.mathworks.com/matlabcentral/fileexchange/170296-enzyme-action-optimizer-a-novel-bio-inspired-optimization and PYTHON at: https://github.com/AliRodan/Enzyme-Action-Optimizer.

Keywords Metaheuristic · Optimization · Heuristic · Enzyme action optimizer · EAO · Engineering design problems · Particle swarm optimization · Swarm intelligence optimization · Artificial intelligence · Global optimization

1 Introduction

Metaheuristics is a class of algorithms designed to solve complex optimization problems that are challenging to traditional methods. These techniques are important for addressing problems characterized by large, complex, and highly nonlinear search spaces, where traditional optimization approaches are often

Extended author information available on the last page of the article ineffective due to computational complexity [1]. Metaheuristics represent high-level procedures that guide heuristic searches to effectively explore and exploit the solution landscape. Moreover, they aim to find optimal solutions within a reasonable timeframe. Many metaheuristic algorithms are inspired by natural processes and phenomena, providing robust mechanisms for exploring large and complex solution spaces [2].

The landscape of metaheuristic techniques is diverse, containing a broad spectrum of algorithms inspired by natural phenomena, psychological processes, and even human-made systems [3]. The most widely used among these are evolutionary algorithms (EAs) [4], including differential evolution (DE) [5], swarm intelligence (SI) [6], which includes algorithms like particle swarm optimization (PSO) [7] and ant colony optimization (ACO) [8], and physics-based methods like the gravitational search algorithm (GSA) [9]. Each of these algorithms operates on the principle of iterative improvement, where a set of candidate solutions goes through various processes such as selection, crossover, mutation, movement, and updating of positions, to converge toward an optimal solution. [10].

Metaheuristic techniques are applied across numerous domains, including engineering design optimization, financial modeling, logistics and supply chain management, bioinformatics, machine learning, and network design [3, 11]. Their adaptability and robustness in handling multimodal, nonlinear, and high-dimensional problems make them a highly effective approach for both research and practical applications [12].

However, metaheuristics come with several limitations and challenges, including the risk of premature convergence, the trade-off between exploration and exploitation, and the need for parameter tuning. Additionally, the 'No-Free-Lunch' theorem [13] states that no single optimization algorithm is universally optimal for all problems. Therefore, selecting a suitable metaheuristic algorithm should be guided by the specific nature, characteristics, and requirements of the problem. This highlights the need to develop new optimization algorithms capable of effectively addressing the diverse range of optimization challenges addressed in engineering and other domains [14].

The objective of this paper is to introduce a novel bio-inspired optimization algorithm, enzyme action optimization (EAO), inspired by the adaptive mechanisms of enzymatic actions in biological systems. EAO is designed to tackle complex, multi-dimensional optimization challenges, including those in engineering and real-world applications. Drawing from the adaptive and catalytic mechanisms of biological enzymes, which are widely recognized for their efficiency in guiding substrates toward optimal states even under varying environmental conditions [15, 16], EAO mimics these principles by rapidly exploring complex search spaces and dynamically adjusting its exploration and exploitation strategies based on observed performance. This biologically inspired design enables EAO to handle diverse problem types and adapt efficiently to various optimization landscapes, reflecting the adaptability and resilience inherent in metabolic processes [16].

Beyond its biological foundation, EAO is designed to address significant challenges in complex optimization scenarios. Two primary computational problems motivate its development. First, high-dimensional optimization often leads to the so-called curse of dimensionality, where the extensive search space prevents classical optimizers from achieving efficient and reliable convergence. Second, real-world objective functions frequently exhibit multimodality, discontinuities, or noisy gradients, further complicating the search process [3]. EAO tackles these challenges by maintaining a population of substrate candidates that collectively explore multiple regions of the decision space. It incorporates stochastic elements to escape local minima, enabling a cooperative mechanism that balances focused exploration of promising solutions with broader exploration of complex regions in the search space.

By effectively addressing these challenges, EAO highlights its adaptability and robust problem-solving capabilities. Its design enables it to tackle a diverse range of optimization problems, positioning it as a promising tool for engineering and other real-world applications.

The main contributions of this paper are summarized as follows:

- Enzyme action optimizer (EAO): A novel bio-inspired optimization algorithm designed for solving complex and engineering design problems.
- Biological inspiration: EAO simulates the adaptive mechanisms of enzymatic actions in biological systems.
- Comprehensive benchmarking: EAO was evaluated on diverse benchmark datasets, including the 23 classical benchmark functions, IEEE CEC2017 and CEC2022 benchmark functions, and five engineering design problems.
- Performance superiority: EAO outperforms 14 state-of-the-art, recent, and highly cited optimization algorithms.

The rest of this paper is organized as follows: Section 2 reviews related work, discussing existing optimization algorithms and categorizing them based on their inspiration. Section 3 introduces the enzyme action optimization (EAO) algorithm, detailing its biological foundation, mathematical model, and exploration-exploitation strategies. Section 4 describes the experimental setup, including benchmark descriptions, compared optimizers, evaluation metrics, and parameter settings. Section 5 presents a comparative analysis of EAO against state-of-the-art optimizers across various datasets. Section 6 examines EAO’s sensitivity, search history, trajectory, and average fitness, along with convergence analysis and an evaluation of its exploration and exploitation capabilities. Section 7 explores EAO’s application to engineering design problems. Finally, Section 8 concludes the paper by summarizing key findings and suggesting future research directions.

2 Related work

Optimization algorithms play a crucial role in computational research, providing powerful tools to solve complex problems across diverse domains. Many of these algorithms are inspired by natural and social phenomena, translating biological, physical, chemical, and behavioral principles into effective computational strategies [3, 10, 17]. Their diversity derives from a wide range of inspirations, including animal behaviors, plant interactions, and physical processes [3]. This broad

Table 1 Summary of state-of-the-art optimization algorithms

Category	Algorithm	Year	Authors	Brief description	Ref.
Swarm intelligence	Particle swarm optimization (PSO)	1995	Kennedy, Eberhart	Models swarm behavior of birds/fish; particles adjust velocity and position based on personal and global best solutions	[18]
	Ant colony optimization (ACO)	2006	Dorigo, Stützle	Simulates foraging behavior of ants; pheromone trails guide selection of promising paths in a graph	[19]
	Artificial bee colony (ABC)	2005	Karaboga, Basturk	Mimics bee foraging patterns; emphasis on shared knowledge of nectar-rich sources	[20]
	Whale optimization algorithm (WOA)	2016	Mirjalili, Lewis	Simulates humpback whales' bubble-net feeding; spiral updating surrounds prey (best solutions)	[52]
	Bacteria phototaxis optimizer (BPO)	2023	Pan, Tang, Zhan, Li	Inspired by bacterial phototaxis; uses photosensory proteins, phototaxis motion, and growth operators to balance exploration and exploitation	[53]
Mammalian behavior	Gray wolf optimizer (GWO)	2014	Mirjalili et al.	Mimics gray wolf structure (alpha, beta, delta, omega) to direct exploration and exploitation	[13]
	Harris hawks optimizer (HHO)	2019	Heidari, Mirjalili	Uses covert attack strategies of hawks; dynamic jump strategies enhance diversity	[25]
	Cheetah optimizer (CO)	2022	Akbari, Deb	Inspired by cheetah chase speed; rapid broad search transitions to more precise exploitation near optima	[23]
	Meerkat optimization algorithm (MOA)	2023	Xian et al.	Mimics meerkats' alertness; group coordination refines solution searching	[24]
Physical phenomena	Simulated annealing (SA)	1983	Kirkpatrick et al.	Based on thermal annealing; cooling schedule enables occasional acceptance of worse solutions	[26]
	Gravitational search algorithm (GSA)	2009	Rashedi et al.	Considers candidate solutions as masses; gravitational Force draws solutions toward fitter agents	[27]
	Multi-verse optimizer (MVO)	2016	Mirjalili et al.	Inspired by multi-verse theory; objects migrate among universes with varied inflation rates	[28]
	Black hole algorithm (BH)	2013	Hatamlou	Analogous to a black hole attracting stars; solutions converge by "falling" into gravity wells	[29]
	Elastic deformation optimization algorithm (EDOA)	2022	Pan, Tang, Lao	Based on Hooke's law of elasticity and Newton's second law; adapts elastic forces to balance exploration and exploitation	[54]

Table 1 continued

Category	Algorithm	Year	Authors	Brief description	Ref.
Chemical biochemical	Atom search optimization (ASO)	2019	Zhao et al.	Simulates interatomic forces; solutions find equilibrium under attractive-repulsive dynamics	[30]
	Chemical reaction optimization (CRO)	2009	Lam, Li	mimic molecular reactions; combination and decomposition events seek stable, lower-energy solutions	[31]
	Nuclear reaction optimization (NRO)	2019	Wei et al.	Based on nuclear fission/fusion; maintains population diversity and enhances search	[32]
Evolutionary algorithms	Differential evolution (DE)	1997	Storn, price	Creates new candidates by adding scaled differences among existing solutions	[5]
	Genetic algorithm (GA)	1975	Holland	Employs survival-of-the-fittest principles, along with crossover and mutation operators, to evolve a population toward optimal solutions	[33]
	Genetic programming (GP)	1998	Banzhaf et al.	Evolves computer programs represented as trees; genetic operators adapt code structures	[34]
Mathematical models	Arithmetic optimization algorithm (AOA)	2021	Abualigah et al.	Employs arithmetic operations; transitions from broad exploration to focused exploitation	[35]
	Chaos game optimization (CGO)	2021	Talatahari et al.	Uses chaos and fractals; random iteration strategies support varied search paths	[36]
	Sine cosine algorithm (SCA)	2016	Mirjalili	Updates solutions via sine/cosine functions; balances global and local search	[37]
	Expectation-based influence evaluation in weighted hypergraphs (EIOA)	2024	Pan, Wang et al.	influence maximization in weighted hypergraphs; incorporates the adaptive dissemination model to evaluate influence spread	[55]
Environmental ecological	Snow ablation optimizer (SAO)	2023	Deng et al.	Simulates snow melting; iterative ablation process reveals promising regions	[38]
	Water cycle algorithm (WCA)	2012	Eskandar et al.	Models water flow; streams merge into rivers and lakes at optimal reservoirs	[39]
Human behavior / Social dynamics	Chef-based optimization algorithm (CBOA)	2022	Trojovská et al.	Refines solutions like iterative recipe testing; adjustments guided by continuous feedback	[40]
	Cultural algorithms (CA)	1994	Reynolds	Integrates individual adaptation with a shared belief space for collective learning	[41]

Table 1 continued

Category	Algorithm	Year	Authors	Brief description	Ref.
Game-based algorithms	Darts game optimizer (DGO)	2020	Dehghani et al.	Inspired by dart throwing; solutions adapt by aiming closer at target optima iteratively	[42]
	Puzzle optimization algorithm (POA)	2022	Ahmadi et al.	Analogous to solving a puzzle; piecewise adjustments assemble coherent optimal structures	[43]
Economic theory	Supply–demand-based optimization (SDO)	2019	Zhao et al.	Mimic market equilibrium; supply–demand interactions converge on best solutions	[45]
	Search and rescue optimization (SAR)	2020	Shabani et al.	Models rescue missions; systematic hunts locate high-fitness solutions (survivors)	[46]

foundation contributes to the development of algorithms that are not only effective but also robust and adaptable to the complex and multi-dimensional challenges presented in optimization tasks. Each algorithm is designed to mimic specific aspects of nature, offering unique strategies for discovering efficient solutions in multi-dimensional and dynamic optimization landscapes [10]. By modeling natural phenomena, these algorithms optimize functions through exploration, exploitation, and evolution, reflecting the mechanisms observed in natural systems [3]. Accordingly, Table 1 provides a detailed classification of state-of-the-art optimization algorithms, categorizing them based on their primary inspiration and core methodologies.

Swarm-inspired algorithms utilize the collective intelligence and decentralized decision-making observed in social animals. For example, particle swarm optimization (PSO) [18] is inspired by the foraging behavior of bird flocks and fish schools. It maintains a population of particles (candidate solutions) that navigate the search space by updating their velocities and positions, guided by both individual and collective best-performing solutions. Over iterations, the swarm converges toward high-quality regions of the search space. Similarly, ant colony optimization (ACO) [19] employs pheromone deposition and evaporation to reinforce promising paths, balancing the exploitation of known routes with the exploration of new possibilities. Several other swarm-based approaches, including artificial bee colony (ABC) [20], ant lion optimizer (ALO) [21], and jellyfish search (JS) [22], demonstrate the diversity of swarming behaviors that can be utilized for optimization tasks.

Some metaheuristics are inspired by mammalian behavior, incorporating foraging strategies, social structures, or alertness mechanisms. The gray wolf optimizer (GWO) [13] mimics the hierarchical structure of wolf packs, while the cheetah optimizer (CO) [23] replicates the rapid chases and sudden directional changes of cheetahs. Similarly, the meerkat optimization algorithm (MOA) [24] is based on meerkats' coordinated alertness, and the Harris hawks optimizer (HHO)

[25] models hawks' sudden attack strategies. These nature-inspired behaviors enhance search diversity while guiding solutions toward high-quality regions.

Another broad category of optimization algorithms is inspired by physical and chemical phenomena. Simulated annealing (SA) [26], for example, is based on metallurgical annealing, incorporating a temperature parameter that gradually decreases to allow temporary acceptance of worse solutions, helping escape local optima. Similarly, the gravitational search algorithm (GSA) [27] models candidate solutions as masses subject to gravitational attraction, where fitter (heavier) solutions apply stronger pull, guiding the population toward optimal solutions. The multi-verse optimizer (MVO) [28] simulates inflation rates across parallel universes, while the black hole algorithm (BH) [29] represents solutions converging into a dominant gravitational well.

In the chemical and biochemical domain, atom search optimization (ASO) [30], chemical reaction optimization (CRO) [31], and nuclear reaction optimization (NRO) [32] utilize molecular interactions, reaction dynamics, and fission or fusion processes to navigate the search space toward energetically favorable configurations.

Evolutionary algorithms (EAs) derive their mechanisms from biological evolution. Genetic algorithms (GA) [33] use selection, crossover, and mutation operators to iteratively refine a population of solutions. By favoring fitter solutions while introducing controlled randomness, GAs maintain diversity and reduce the risk of premature convergence. Over successive generations, the population gradually moves toward global or near-global optima. Differential evolution (DE) [5] optimizes solutions by applying scaled differences between existing candidates, while genetic programming (GP) [34] extends these principles to evolve entire programs, demonstrating the adaptability and versatility of evolution-based optimizers.

Mathematical and numerical approaches are another category of optimization algorithms, relying on pure mathematical principles rather than biological or social analogies. For instance, the arithmetic optimization algorithm (AOA) [35] applies arithmetic operators such as addition, subtraction, multiplication, and division in a controlled manner. It begins with broad exploration and gradually narrows its focus on promising regions, with an adjustable parameter modulating the intensity of arithmetic operations over time. Similarly, chaos game optimization (CGO) [36] and the sine cosine algorithm (SCA) [37] apply fractal geometry and trigonometric models to systematically balance exploration and exploitation within the search space.

Environmental and ecologically inspired methods draw inspiration from natural cycles and environmental dynamics to enhance the search process. The snow ablation optimizer (SAO) [38] models snow melting processes, while the water cycle algorithm (WCA) [39] simulates the natural water cycle, including evaporation, precipitation, and flow. By modeling ecological cycles, these algorithms achieve a dynamic balance between exploration and exploitation.

Human social behavior has also inspired optimization algorithms. The chef-based optimization algorithm (CBOA) [40] simulates the decision-making strategies of chefs in recipe creation and refinement. Similarly, cultural algorithms (CA) [41] integrate individual learning with a shared belief space, enabling collective adaptation and knowledge exchange across generations.

Further exploring non-biological sources, game-based optimizers such as the darts game optimizer (DGO) [42], puzzle optimization algorithm (POA) [43], and game of squid optimizer (SGO) [44] integrate strategic decision-making principles from recreational activities and popular culture. Similarly, economics-inspired algorithms like supply–demand-based optimization (SDO) [45] and search and rescue optimization (SAR) [46] utlize market equilibrium dynamics and systematic rescue strategies to converge on optimal solutions.

As highlighted in Table 1, the extensive diversity of optimization algorithms spans multiple domains, including swarm intelligence, mammalian hunting strategies, and physical, chemical, and mathematical analogies. This broad spectrum demonstrates the flexibility and adaptability of metaheuristics in tackling complex optimization challenges.

Despite their diversity, existing metaheuristic and evolutionary algorithms often face significant limitations, including premature convergence, slow adaptation to dynamic landscapes, and extensive parameter tuning requirements [3, 47]. Traditional evolutionary approaches frequently exhibit exponential growth in computational cost, making them inefficient for handling extremely large search spaces. Additionally, algorithms dependent on strict gradient information struggle with highly nonlinear or discontinuous problems, highlighting the need for flexible, gradient-free techniques [3].

These challenges significantly impact the performance of metaheuristics, particularly in high-dimensional and complex optimization tasks. One of the most critical issues is premature convergence, where algorithms fail to maintain solution diversity, particularly in multimodal landscapes [48]. This often causes premature convergence to local optima, limiting their ability to identify global solutions [49]. Another major challenge is slow adaptation to dynamic landscapes, as many algorithms rely on fixed exploration and exploitation mechanisms [50]. This rigidity reduces their efficiency in responding to time-varying objectives and constraints, making them less effective in real-world applications [51].

Moreover, extensive parameter tuning presents a significant barrier to widespread applicability. Parameters such as mutation rates, crossover probabilities, and weight scaling often require manual adjustments [51], a time-consuming process that weakens generalizability and practical usability. However, as problem dimensionality increases, computational costs rise exponentially due to the “curse of dimensionality.” Traditional evolutionary approaches require more iterations and evaluations, reducing scalability for large-scale problems [48]. Metaheuristics also struggle with highly nonlinear, discontinuous, or noisy objective functions, where gradient-based methods fail to provide reliable solutions. This highlights the necessity for robust, gradient-free techniques that can handle complex real-world problems [50].

Additionally, these algorithms lack robustness in multimodal optimization, where multiple local optima exist [50]. Many fail to explore and exploit multiple search regions simultaneously, resulting in suboptimal solutions when fitness landscapes contain significant gaps. Scalability remains a persistent challenge, as many algorithms are optimized for smaller-scale problems but perform poorly as complexity increases [49].

Lastly, heavy dependence on problem-specific modifications limits the general applicability of metaheuristics [48]. Many require extensive customization for different problem domains, reducing their versatility across a broad range of optimization challenges.

To address these challenges, the enzyme action optimizer (EAO) introduces enzyme-inspired mechanisms that systematically reduce the likelihood of premature convergence. By modeling candidate solutions as substrates interacting with an enzyme-like process, EAO effectively enhances exploration and exploitation. Key features such as selective binding, cofactor-driven adjustments, and stochastic fluctuations mimic complex biological dynamics, improving adaptability across diverse optimization landscapes. EAO incorporates adaptive coefficients and catalytic updating schemes, extending the capabilities of bio-inspired algorithms. These mechanisms maintain solution diversity while ensuring convergence, making EAO particularly effective in noisy and multimodal problem spaces.

A significant advantage of EAO is its simplicity, requiring only one primary parameter: enzyme concentration. Empirical studies suggest that this parameter, typically set to a small value, maintains an optimal balance between exploration and exploitation. Additionally, the adaptive factor (AF) dynamically adjusts the search process, stabilizing performance compared to other state-of-the-art optimizers. These attributes make EAO particularly suitable for complex, high-dimensional, and multimodal optimization problems.

By utilizing enzyme-inspired principles, EAO effectively overcomes the core limitations of existing metaheuristic approaches, offering a robust, biologically motivated framework for complex optimization challenges. The following section details EAO's formulation and its application in engineering design problems.

3 Enzyme action optimizer (EAO)

3.1 Inspiration

EAO draws its inspiration from the catalytic behavior of enzymes in biological systems. Enzymes are specialized proteins that accelerate chemical reactions by binding to target molecules, known as substrates [16]. Upon binding, the enzyme–substrate complex experiences subtle structural changes that lower the activation energy and increase the reaction rate without consuming the enzyme [15]. This adaptive mechanism, in which the enzyme (catalyst) facilitates substrate transformation under the influence of cofactors and environmental signals, forms the basis of the EAO optimization algorithm.

In enzyme kinetics, cofactors, pH levels, temperature, and concentration gradients can all influence substrate transformation [56]. Enzymes adopt diverse configurations and use flexible active sites, balancing exploration and exploitation of metabolic pathways [56]. Similarly, EAO parallels these adaptive steps by employing an enzyme–substrate analogy, wherein candidate solutions (substrates)

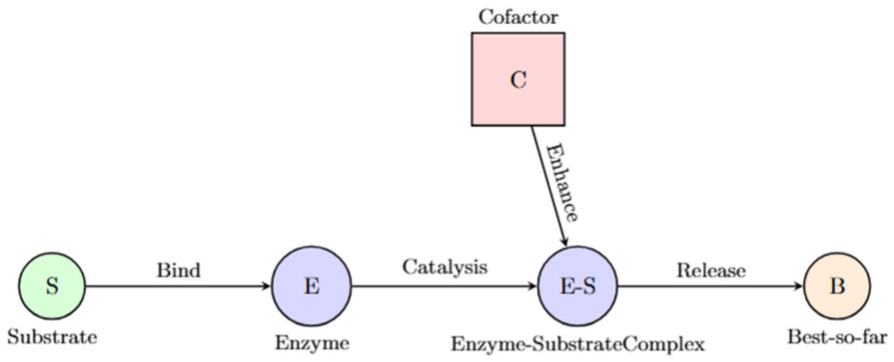

Fig. 1 Illustration of the enzyme mechanism

interact with a best-so-far solution (enzyme). Random disturbances reflect the stochastic dynamics of biological processes, as illustrated in Fig. 1.

Moreover, many enzymes rely on non-protein molecules (e.g., metal ions or coenzymes) to enhance or enable catalytic activity [57]. In EAO, this idea translates into cofactor-inspired control parameters, such as adaptive factor (AF) and enzyme concentration (EC), which regulate exploration and exploitation. These parameters mimic the self-regulatory nature of enzyme systems, ensuring sufficient population diversity (exploration) while intensifying convergence (exploitation) around promising solutions.

By incorporating selective substrate binding, cofactor-driven adjustments, and stochastic fluctuations [57], EAO captures the core of biochemical catalysis. This synergy of broad exploration and focused refinement reflects how enzymes effectively drive metabolic processes in noisy and uncertain environments [16]. Figure 2 provides a schematic illustration of the enzyme-inspired workflow in EAO, from substrate activation through the enzyme–cofactor complex to a final solution (product). This process reflects the natural catalytic pathway, showing how EAO transitions from a broad set of potential solutions to more refined outcomes, similar to enzymes binding and transforming substrates.

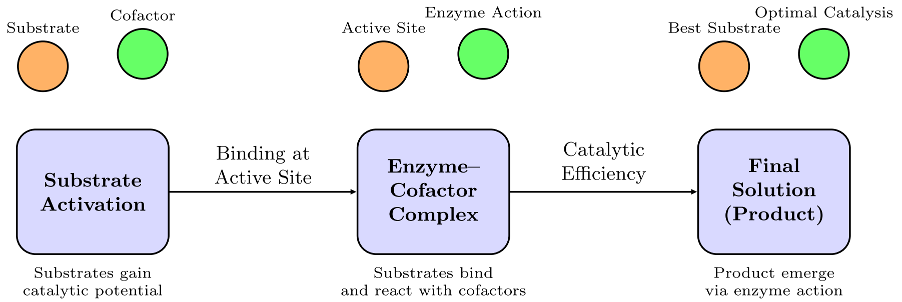

Fig. 2 A biology-inspired schematic for enzyme action optimization (EAO)

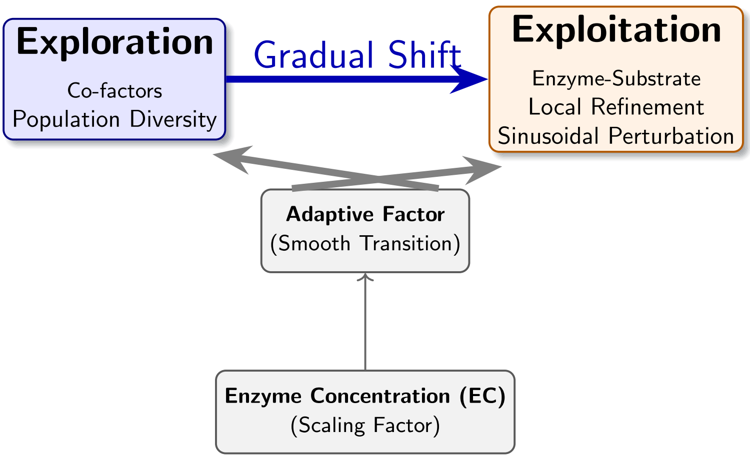

Fig. 3 Gradual shift between exploration and exploitation in EAO

Additionally, to guide solution discovery and improvement, EAO explicitly models a catalytic interaction between the best solution (enzyme) and the remaining candidate solutions (substrates). This interaction is modulated by adaptive parameters, primarily the adaptive factor (AF) and enzyme concentration (EC), which reflect cofactor-mediated and environmentally influenced changes in real enzymes.

As illustrated in Fig. 3, the adaptive factor (AF) ensures a smooth transition from exploration to exploitation, progressively shifting the search behavior. Initially, cofactors promote exploration by enhancing population diversity, while EC regulates how strongly substrates are influenced by the best solution, balancing adaptation across iterations. In later stages, enzyme-substrate interactions, local refinement, and sinusoidal perturbations intensify exploitation, guiding solutions toward promising regions of the search space.

3.2 Mathematical model of EAO

This section explains the mathematical model of EAO, where the first step is substrate initialization. The number of search agents (substrates) is denoted by N, the maximum number of iterations by T, and the dimensionality of the search space by dim. Each decision variable is constrained to lie between a lower bound ($\mathbf{LB}$) and an upper bound ($\mathbf{UB}$), represented as vectors that define the feasible region in each dimension. Let $f(\mathbf{x})$ represent the objective function to be minimized. Equation (1) provides the random initialization scheme for the substrate pool:

$$\mathbf{X}_i^{(0)} = \mathbf{LB} + (\mathbf{UB} - \mathbf{LB}) \odot \mathbf{r}_i, \tag{1}$$

where $\mathbf{X}_i^{(0)}$ is the initial position of the i-th search agent (substrate), $\mathbf{r}_i$ is a vector of uniformly distributed random values in [0, 1], and $\odot$ denotes elementwise multiplication. The index i runs from 1 to N.

Once the initial substrate positions have been generated, Eq. (2) is used to evaluate each substrate's fitness and thus measure its quality:

$$F_i^{(0)} = f\left(\mathbf{X}_i^{(0)}\right), \tag{2}$$

where $F_i^{(0)}$ is the initial fitness of the i-th substrate.

The best substrate at iteration 0 is determined by identifying the minimal value of $F_i^{(0)}$, and the corresponding position is defined as $\mathbf{X}_{\text{best}}^{(0)}$. The quantity $F_{\text{best}}^{(0)} = f(\mathbf{X}_{\text{best}}^{(0)})$ denotes the best fitness value at iteration 0. The main iterative loop runs from $t = 1$ to $t = T$, during which several adaptive strategies are employed. Equation (3) defines the adaptive factor (AF) for each iteration t:

$$\mathrm{AF_t} = \sqrt{\frac{\mathrm{t}}{\mathrm{MaxIter}}}. \tag{3}$$

This square root relationship ensures a gradual increase in the adaptive factor as the iterations progress.

At each iteration t, every substrate $\mathbf{X}_i^{(t-1)}$ generates two candidate positions. Equation (4) describes the first substrate candidate position:

$$\mathbf{X}_{i,1}^{(t)} = \left(\mathbf{X}_{\text{best}}^{(t-1)} - \mathbf{X}_i^{(t-1)}\right) + \boldsymbol{\rho}_i \odot \sin\left(\mathrm{AF_t} \cdot \mathbf{X}_{\mathrm{i}}^{(\mathrm{t}-1)}\right), \tag{4}$$

where $\boldsymbol{\rho}_i$ is a random vector in $[0, 1]^{\text{dim}}$, and the sine function is applied element-wise. The resulting position $\mathbf{X}_{i,1}^{(t)}$ is constrained to the interval $[\mathbf{LB}, \mathbf{UB}]$ to ensure it remains within the feasible region.

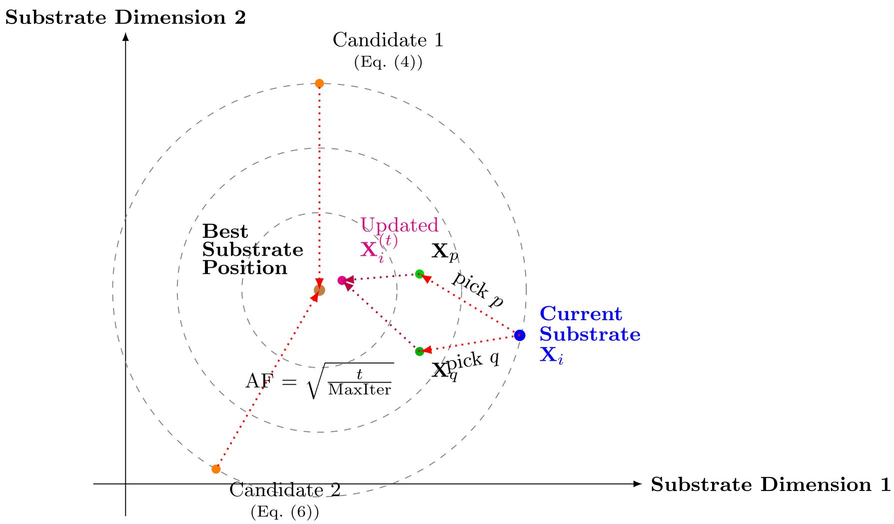

Fig. 4 Conceptual model of best substrate position update

To generate the second substrate candidate position, two distinct substrate indices p, q are selected from the set $\{1, 2, \ldots, N\}$. These indices define the vector $\mathbf{d}$ according to Eq. (5):

$$\mathbf{d} = \mathbf{X}_p^{(t-1)} - \mathbf{X}_q^{(t-1)}, \tag{5}$$

where this difference serve to inject additional diversity into the search process.

Equation (6) defines the second substrate candidate position. It is constructed by combining the element introduced above:

$$\mathbf{X}_{i,2}^{(t)} = \mathbf{X}_i^{(t-1)} + \mathrm{sc}_1\, \mathbf{d}_1 + \mathrm{AF_t}\, \mathrm{sc}_2 \left(\mathbf{X}_{\mathrm{best}}^{(\mathrm{t}-1)} - \mathbf{X}_{\mathrm{i}}^{(\mathrm{t}-1)} \right) \tag{6}$$

The coefficients sc_1 and sc_2 are random scale factors in the interval $[\mathrm{EC}, 1]$, where EC is the enzyme concentration (usually a small constant, such as 0.1). This position is also constrained within $[\mathbf{LB}, \mathbf{UB}]$ to preserve feasibility.

At each iteration t, both $\mathbf{X}_{i,1}^{(t)}$ and $\mathbf{X}_{i,2}^{(t)}$ are evaluated, and the best position of these two substrates is chosen as shown by Eq. (7)

$$\mathbf{X}_{i,\mathrm{upd}}^{(t)} = \begin{cases} \mathbf{X}_{i,1}^{(t)}, & \text{if } F(\mathbf{X}_{i,1}^{(t)}) < F(\mathbf{X}_{i,2}^{(t)}), \\ \mathbf{X}_{i,2}^{(t)}, & \text{otherwise.} \end{cases} \tag{7}$$

This updated position replaces $\mathbf{X}_i^{(t-1)}$ when it reach better position as shown by Eq. (8):

$$\mathbf{X}_i^{(t)} = \begin{cases} \mathbf{X}_{i,\mathrm{upd}}^{(t)}, & \text{if } F(\mathbf{X}_{i,\mathrm{upd}}^{(t)}) < F(\mathbf{X}_i^{(t-1)}), \\ \mathbf{X}_i^{(t-1)}, & \text{otherwise.} \end{cases} \tag{8}$$

The global best is then updated as shown in Eq. (9):

$$\text{if } F\left(\mathbf{X}_i^{(t)}\right) < F_{\mathrm{best}}^{(t-1)} \implies \mathbf{X}_{\mathrm{best}}^{(t)} = \mathbf{X}_i^{(t)}, \quad F_{\mathrm{best}}^{(t)} = F\left(\mathbf{X}_i^{(t)}\right). \tag{9}$$

In EAO, each substrate position is kept within its allowable interval by constrained the updated substrate positions to the lower and upper bound vectors, $\mathbf{LB}$ and $\mathbf{UB}$. After each update step (Eqs. (4)–(6)), any dimension d of a substrate $\mathbf{X}_i$ that exceeds the upper bound is set to $\mathbf{UB}_d$, whereas any dimension that goes below the lower bound is set to $\mathbf{LB}_d$, as described in Eq. 10

$$\mathbf{X}_i(d) = \max\Big(\min\big(\mathbf{X}_i(d), \mathbf{UB}_d\big), \mathbf{LB}_d\Big), \quad \forall d \in \{1, \ldots, \dim\}. \tag{10}$$

This mechanism guarantees that all solutions remain within the search space bounds for every iteration.

To better illustrate the update mechanism in EAO, Fig. 4 offers a conceptual layout of the search space showing how these candidate updates revolve around the current substrate $\mathbf{X}_i$, the difference vector $\mathbf{d}$, and the best substrate.

3.3 EAO algorithm analysis

Algorithm 1 outlines EAO in pseudocode. The process begins by initializing a pool of substrates (candidate solutions) (Step 1) and identifying the best solution. Then, for each iteration (Step 2), a control factor ($\mathrm{AF_t}$) is computed. Every substrate produces two candidate positions:

1. A position partially informed by the current best substrate.
2. A position influenced by the difference between two randomly chosen substrates, injecting population diversity.

After evaluation, the better candidate replaces the original substrate if it yields improved fitness. The global best solution is updated accordingly. Upon reaching *T* iterations, the final best substrate is returned as the approximate global optimum (Step 3).

Algorithm 1 Enzyme action optimizer (EAO)

Require: N (number of substrates), T (max iterations), **LB**, **UB**, dim, $f(\cdot)$ (objective function)

Step 1: Initialization
Generate the substrate pool $\{\mathbf{X}_i^{(0)}\}_{i=1}^{N}$ Eq. (1)
for $i = 1 \to N$ **do**
    Evaluate $F_i^{(0)} \leftarrow f\big(\mathbf{X}_i^{(0)}\big)$ Eq. (2)
end for
Determine $\mathbf{X}_{\mathrm{best}}^{(0)}$ as the substrate with the minimum $F_i^{(0)}$; let $F_{\mathrm{best}}^{(0)} \leftarrow f\big(\mathbf{X}_{\mathrm{best}}^{(0)}\big)$

Step 2: Main Evolution Loop
for $t = 1 \to T$ **do**
    Compute AF_t using Eq. (3)
    for $i = 1 \to N$ **do**
        (a) First Substrate Position
          Calculate $\mathbf{X}_{i,1}^{(t)}$ using Eq. (4)
          $F_{i,1}^{(t)} \leftarrow f\big(\mathbf{X}_{i,1}^{(t)}\big)$
        (b) Second Substrate Position
          Pick 2 distinct substrate (p, q) Eq. (5)
          Calculate $\mathbf{X}_{i,2}^{(t)}$ Eq. (6)
          $F_{i,2}^{(t)} \leftarrow f\big(\mathbf{X}_{i,2}^{(t)}\big)$
        (c) Selection and Update
          Select best position Eqs. (7)– (8)
          Update global best Eq. (9)
    end for
end for

Step 3: Output
Return $(\mathbf{X}_{\mathrm{best}}^{(T)})$

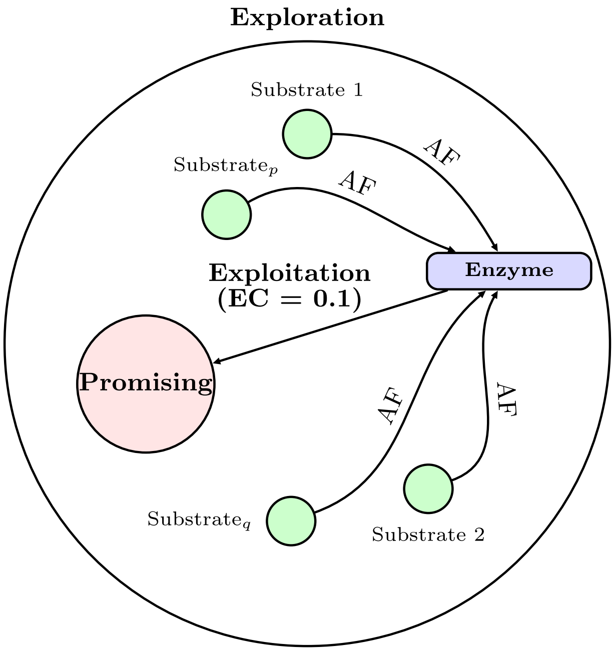

Fig. 5 Exploration and exploitation behavior in EAO

3.4 Exploration and exploitation behavior in EAO

EAO maintains a dynamic balance between exploration (searching diverse regions) and exploitation (refining promising areas), reflecting how enzymes in nature adapt catalytic rates and substrate preferences. Figure 5 conceptually illustrates these dual mechanisms:

Exploration in EAO:

- **Population diversity**: Multiple substrates traverse different regions of the search space.
- **Adaptive factor shifts**: AF_t starts at lower values (favoring exploration) and increases gradually, driving the algorithm toward exploitation in later stages.
- **Difference-based perturbations**: Randomly selected substrate indices (p, q) add fresh directions to avoid premature convergence.

Exploitation in EAO:

- **Best-substrate guidance**: Substrates update around the current best, focusing on promising areas.

- **Sinusoidal perturbations**: Sine-based updates (Eq. (4)) refine local searches, balancing exploitation and exploration.
- **Selection mechanism**: Only improvements replace existing substrates, ensuring continuous progress.
- **Step size control via enzyme concentration (EC)**: The small EC value (typically 0.1) acts as a stabilizing factor, regulating step size during exploitation to prevent excessive movement near promising solutions.

By transitioning from broad, diverse sampling in the early iterations to refined local searches in later phases, EAO effectively avoids premature convergence and accelerates toward global or near-global optima.

3.5 EAO computational complexity

The computational complexity of EAO can be analyzed based on its three primary computational stages: initialization, reaction (iteration) evaluation, and iterative updates. Each of these stages contributes differently to the overall computational demand of EAO.

The initialization of the substrate matrix S of size $n \times d$, where n is the number of substrates and d is the dimensionality of each substrate, involves setting up nd entries with random values within the specified bounds. This step is executed once and has a complexity of:

$$O(nd) \tag{11}$$

During each iteration of EAO, every substrate S_i is evaluated to determine its efficiency. Assuming the evaluation function has a computational complexity of $O(F)$, where F depends on the substrate evaluation, the evaluation complexity for all substrates once per iteration is:

$$O(nF) \tag{12}$$

Given that there are M reaction, where M is the maximum number of reactions, the total complexity for all evaluations across all reactions becomes:

$$O(MnE) \tag{13}$$

Each reaction involves updating each substrate using high-dimensional and low-dimensional strategies. The computation for each update assumes that the evaluation cost is dominated by the substrate's dimensionality d, as:

$$O(d) \tag{14}$$

Thus, for n substrates per reaction, the complexity is:

$$O(nd) \tag{15}$$

Over M reaction, the complexity for all modifications is:

$$O(Mnd) \tag{16}$$

The total computational complexity of EAO is the sum of the complexities of initialization, evaluation, and iterative updating processes:

$$O(nd) + O(MnF) + O(Mnd) = O(MnF + Mnd) \tag{17}$$

4 Experimental setup

This section includes an overview of the experimental setup, which includes a description of the benchmark functions, the optimizers used for comparison, evaluation metrics, and parameter settings.

4.1 Benchmark description

The 23 Classical benchmark functions: These functions consist of a diverse range of problems that contain unimodal, multimodal, and fixed-dimension multimodal functions. Unimodal functions (F1-F7), as shown in Appendix B. Table 22. The multimodal functions (F8-F13) are shown in Appendix B. Table 23 introduce multiple optima, challenging the algorithms to explore effectively and avoid local minima. It contains functions that involve sinusoidal and exponential components to create complex landscapes. Finally, the fixed-dimension multimodal functions (F14-F23), as shown in Appendix B. Table 24 presents real-world inspired challenges with very specific constraints that simulate engineering and design problems, highlighting the ability of algorithms to handle practical applications with precision.

IEEE CEC2017: These benchmark functions are part of the IEEE congress on evolutionary computation and consist of a diverse set of test functions designed to evaluate the performance of optimization algorithms. This dataset includes 30 functions categorized into several types: unimodal, multimodal, hybrid, and composition functions, each presenting unique challenges as shown in Appendix C. Table 25. The unimodal functions tested the exploitation capabilities of the algorithms, multimodal functions examined their exploration skills in avoiding local optima, hybrid functions combined features of the first two types to test both capabilities simultaneously, and composition functions provided a complex landscape by composing several basic functions. This collection aims to provide a comprehensive assessment of an algorithm's ability to effectively navigate and optimize complex, high-dimensional spaces across a variety of optimization problems.

IEEE CEC2022: This benchmark is designed for the IEEE Congress on Evolutionary Computation and features a set of test functions intended to evaluate and challenge various aspects of evolutionary algorithms. This benchmark typically covers a range of problem types, including unimodal, multimodal, hybrid, and composition problems, each presenting unique difficulties to test the robustness and adaptability as shown in Appendix C. Table 26. This collection of functions

Table 2 List of 14 compared optimization algorithms

Algorithm	Proposed by	Year
Horned lizard optimization algorithm (HLOA)	Peraza-Romero et al. [58]	2024
Sea-horse optimizer (SHO)	Zhao et al. [59]	2023
Geometric mean optimizer (GMO)	Rezaei et al. [60]	2023
White shark optimizer (WSO)	Braik et al. [61]	2022
Artificial gorilla troops optimizer (GTO)	Abdollahzadeh et al. [62]	2021
Flow direction algorithm (FDA)	Karami et al. [63]	2021
Henry gas solubility optimization (HGSO)	Hashim et al. [64]	2019
Whale optimization algorithm (WOA)	Mirjalili et al. [52]	2016
Multi-verse optimizer (MVO)	Mirjalili et al. [28]	2016
Sine cosine algorithm (SCA)	Mirjalili et al. [37]	2016
Moth-flame optimization (MFO)	Mirjalili et al. [65]	2015
Grey wolf optimizer (GWO)	Mirjalili et al. [13]	2014
Differential evolution (DE)	Storn and Price [5]	1997
Particle swarm optimization (PSO)	Kennedy and Eberhart [18]	1995

includes tests for scalability, the ability to escape local optima, and handling high-dimensional search spaces.

4.2 Compared optimizers

The performance of EAO was evaluated and compared with a diverse set of 14 recent and highly cited optimization algorithms that are summarized in Table 2.

4.3 Evaluation metrics

The efficiency of each algorithm was statistically measured using the mean and the standard deviation. These metrics were chosen to provide insights into both the average performance and the consistency of the optimization results across multiple runs.

4.4 Parameters settings

To ensure a fair and comprehensive comparison, the substrate/agent count (n) and the Maximum Number of Reactions (M) were set to 30 and 500, respectively, for the 23 classical benchmark functions across all algorithms. Each algorithm was executed 30 times per test function to obtain statistically significant results.

For the evaluation using the IEEE CEC2017 benchmark functions, the substrate/agent count (n) was set to 50, and the maximum number of reactions (M) was increased to 1000 to accommodate the complexity and computational demands of these functions. Additionally, we tested the optimizers on high-dimensional problems ($d = 100$) with an extended number of reactions/iterations (M) of

Table 3 Parameter settings for compared algorithms

Algorithm	Parameter
FDA	Flux coefficient = 0.3, diffusion rate = 0.7
GMO	Geometric rate = 0.01, update frequency = 5
GTO	Tension rate = 0.8, aggression level = 0.2
GWO	Convergence constant a = [2, 0]
HGSO	l1=5E-03, l2=100, l3=1E-02, alpha=1, beta=1, M1=0.1, M2=0.2
HLOA	Does not use additional parameters.
MFO	A linearly decreases from −1 to −2
MVO	WEP_{Max} = 1, WEP_{Min} = 0.2
SCA	r1 = random(0,1), r2 = random(0,1), r3 = random(0,1), r4 = random(0,1)
SHO	U = 0.05, V = 0.05, L = 0.05
WOA	Convergence constant a =[2,0], b=1
WSO	Hunt depth = 0.2, speed factor = 0.8
PSO	Inertia weight $w = 0.5$, cognitive coefficient $c_1 = 1.5$, social coefficient $c_2 = 1.5$
DE	mutation factor $F = 0.5$, crossover rate $CR = 0.9$

10,000. In this case, the substrate/agent count (n) was increased to 100 to ensure sufficient exploration of the high-dimensional search space. This configuration was designed to assess the performance and scalability of all tested optimizers in handling complex and high-dimensional problems.

Finally, for the IEEE CEC2022 benchmark, two distinct experiments were conducted corresponding to different dimensional settings: $d = 10$ and $d = 20$. For these dimensionalities, the maximum number of reactions (M) was set to 10,000 and 20,000, respectively. This setup was adopted to thoroughly explore the solution space and ensure comprehensive optimization outcomes under varying problem complexities.

The specific parameter settings for each algorithm are detailed in Table 3, ensuring transparency and reproducibility of the experimental results.

5 Comparison results of EAO with state-of-the-art optimizers

This section focuses on the performance of EAO compared to state-of-the-art optimizers in various benchmark sets. This includes results on the 23 classical benchmark functions, an in-depth performance analysis on the IEEE CEC2017 benchmark functions (F1-F30) and a detailed discussion of the results on the IEEE CEC2022 benchmark functions (F1-F12).

5.1 Results of the 23 classical benchmark functions

The experimental results presented in Table 4 demonstrate that EAO optimizer maintains superior overall performance compared to the other state-of-the-art

Table 4 Comparison results of the first dataset (23 benchmark functions) across three categories: unimodal (F1–F7), multimodal (F8–F13), and fixed-dimension multimodal (F14–F23)

Function	Stats	EAO	WOA	MFO	MVO	SCA	GWO	WSO
Unimodal Functions (F1–F7)								
F1	Mean	**0.000E + 00**	1.344E-74	2.003E + 03	1.304E + 00	9.485E + 00	3.121E-27	2.714E + 02
	STD	**0.000E + 00**	3.140E-74	4.103E + 03	4.393E-01	2.641E + 01	1.084E-26	1.414E + 02
F2	Mean	**0.000E + 00**	6.388E-51	3.513E + 01	8.764E + 00	9.416E-03	1.128E-16	4.140E + 00
	STD	**0.000E + 00**	1.979E-50	2.001E + 01	2.682E + 01	1.185E-02	7.805E-17	1.269E + 00
F3	Mean	**0.000E + 00**	4.594E + 04	1.882E + 04	2.170E + 02	8.885E + 03	8.847E-06	1.439E + 03
	STD	**0.000E + 00**	1.684E + 04	1.344E + 04	9.284E + 01	4.837E + 03	1.643E-05	5.144E + 02
F4	Mean	**0.000E + 00**	4.212E + 01	7.054E + 01	1.842E + 00	3.900E + 01	5.508E-07	1.299E + 01
	STD	**0.000E + 00**	3.377E + 01	9.276E + 00	7.474E-01	1.237E + 01	5.997E-07	2.724E + 00
F5	Mean	1.947E + 01	2.775E + 01	1.946E + 04	4.217E + 02	2.055E + 05	2.701E + 01	1.781E + 04
	STD	8.583E-01	4.252E-01	3.682E + 04	6.700E + 02	4.946E + 05	7.317E-01	1.195E + 04
F6	Mean	1.077E-06	3.781E-01	1.491E + 03	1.480E + 00	1.733E + 01	8.739E-01	3.163E + 02
	STD	2.594E-07	2.228E-01	4.843E + 03	4.536E-01	1.858E + 01	5.508E-01	1.840E + 02
F7	Mean	4.638E-04	4.193E-03	1.198E + 00	3.605E-02	1.115E-01	1.800E-03	1.383E-01
	STD	3.399E-04	6.037E-03	3.031E + 00	1.572E-02	1.349E-01	9.915E-04	1.122E-01
Multimodal Functions (F8–F13)								
F8	Mean	− 4.825E + 03	**− 1.043E + 04**	− 8.605E + 03	− 7.718E + 03	− 3.812E + 03	− 5.791E + 03	− 6.114E + 03
	STD	4.469E + 02	**1.462E + 03**	9.461E + 02	8.645E + 02	4.208E + 02	4.889E + 02	1.598E + 03
F9	Mean	**0.000E + 00**	**0.000E + 00**	1.733E + 02	1.180E + 02	4.159E + 01	1.040E + 00	4.818E + 01
	STD	**0.000E + 00**	**0.000E + 00**	4.248E + 01	2.483E + 01	3.089E + 01	2.165E + 00	1.287E + 01
F10	Mean	**4.441E-16**	4.086E-15	1.598E + 01	2.089E + 00	1.583E + 01	9.699E-14	5.962E + 00
	STD	**0.000E + 00**	2.276E-15	6.987E + 00	7.294E-01	7.317E + 00	1.853E-14	1.083E + 00
F11	Mean	**0.000E + 00**	**0.000E + 00**	2.341E + 01	8.749E-01	8.861E-01	8.258E-03	2.810E + 00
	STD	**0.000E + 00**	**0.000E + 00**	4.949E + 01	6.407E-02	3.135E-01	1.333E-02	9.615E-01

Table 4 continued

Function	Stats	EAO	WOA	MFO	MVO	SCA	GWO	WSO
F12	Mean	**1.227E-08**	2.225E-02	1.280E + 07	2.701E + 00	2.131E + 03	5.042E-02	3.812E + 00
	STD	**2.994E-09**	1.279E-02	5.724E + 07	1.319E + 00	7.712E + 03	2.944E-02	2.185E + 00
F13	Mean	1.831E-03	5.265E-01	6.103E + 03	1.726E-01	1.206E + 05	6.877E-01	2.187E + 03
	STD	4.165E-03	2.178E-01	2.028E + 04	1.188E-01	5.079E + 05	2.265E-01	5.355E + 03
Fixed-Dimension Multimodal Functions (F14–F23)								
F14	Mean	**9.980E-01**	2.578E + 00	1.788E + 00	9.980E-01	2.084E + 00	3.939E + 00	9.980E-01
	STD	**3.002E-16**	2.137E + 00	1.740E + 00	2.112E-11	2.241E + 00	3.884E + 00	1.676E-15
F15	Mean	**3.075E-04**	7.771E-04	2.426E-03	8.293E-03	9.606E-04	2.519E-03	3.092E-04
	STD	**6.125E-11**	7.437E-04	4.539E-03	1.756E-02	2.897E-04	6.111E-03	7.795E-06
F16	Mean	**−1.032E + 00**	**−1.032E + 00**	**−1.032E + 00**	**−1.032E + 00**	−9.908E-01	**−1.032E + 00**	**−1.032E + 00**
	STD	1.981E-11	8.937E-10	2.278E-16	2.139E-07	1.825E-01	8.467E-09	2.278E-16
F17	Mean	**3.979E-01**	3.979E-01	**3.979E-01**	3.979E-01	**3.979E-01**	**3.979E-01**	**3.979E-01**
	STD	2.567E-09	7.160E-06	**0.000E + 00**	6.464E-07	**0.000E + 00**	**0.000E + 00**	**0.000E + 00**
F18	Mean	**3.000E + 00**	**3.000E + 00**	**3.000E + 00**	**3.000E + 00**	**3.000E + 00**	**3.000E + 00**	**3.000E + 00**
	STD	3.714E-10	1.716E-03	8.033E-10	2.923E-06	3.780E-14	5.612E-09	1.078E-15
F19	Mean	**−3.863E + 00**	−3.859E + 00	**−3.863E + 00**	**−3.863E + 00**	−3.855E + 00	**−3.863E + 00**	**−3.863E + 00**
	STD	**2.782E-03**	3.698E-03	**2.782E-03**	**2.782E-03**	5.580E-03	**2.782E-03**	**2.782E-03**
F20	Mean	**−3.319E + 00**	−3.257E + 00	−3.226E + 00	−3.266E + 00	−2.953E + 00	−3.251E + 00	−3.318E + 00
	STD	**4.655E-03**	6.459E-02	9.457E-02	5.624E-02	3.674E-01	7.096E-02	5.825E-03
F21	Mean	**−1.015E + 01**	−8.792E + 00	−7.137E + 00	−7.709E + 00	−3.690E + 00	−9.398E + 00	**−1.015E + 01**
	STD	3.395E-07	1.362E + 00	3.016E + 00	2.444E + 00	6.463E + 00	7.555E−01	**3.209E−07**
F22	Mean	**−1.040E + 01**	−8.783E + 00	−8.604E + 00	−8.835E + 00	−3.900E + 00	− 1.005E + 01	**−1.040E + 01**
	STD	**1.405E-04**	1.620E + 00	1.799E + 00	1.568E + 00	6.503E + 00	3.539E-01	1.406E-04

Table 4 continued

Function	Stats	EAO	WOA	MFO	MVO	SCA	GWO	WSO
F23	Mean	**−1.054E + 01**	−7.755E + 00	−8.279E + 00	−8.510E + 00	−4.120E + 00	−9.993E + 00	**−1.054E + 01**
	STD	**1.098E-04**	2.782E + 00	2.257E + 00	2.026E + 00	6.416E + 00	5.437E-01	**1.098E-04**
	WIN	**18**	**5**	**4**	**2**	**2**	**4**	**7**

Function	Stats	HGSO	SHO	HLOA	FDA	GTO	GMO	PSO	DE
Unimodal Functions (F1–F7)									
F1	Mean	4.801E-178	4.796E-139	2.415E-228	1.361E-02	**0.000E + 00**	6.246E-164	2.334E-04	1.350E + 01
	STD	0.000E + 00	1.908E-138	0.000E + 00	1.214E-02	**0.000E + 00**	0.000E + 00	3.488E-04	3.236E + 01
F2	Mean	1.056E-76	3.176E-78	1.260E-119	7.853E-03	3.705E-194	4.031E-77	3.008E-02	2.590E-02
	STD	4.724E-76	7.195E-78	5.126E-119	4.346E-03	0.000E + 00	1.801E-76	2.858E-02	6.517E-02
F3	Mean	2.314E-166	4.310E-99	1.217E-227	3.296E + 02	**0.000E + 00**	1.373E-111	8.099E + 01	6.299E + 02
	STD	0.000E + 00	1.300E-98	0.000E + 00	2.703E + 02	**0.000E + 00**	6.045E-111	2.337E + 01	4.501E + 02
F4	Mean	9.546E-86	2.842E-56	1.243E-119	2.154E + 01	6.923E-193	1.292E-84	1.099E + 00	2.577E + 01
	STD	3.086E-85	1.015E-55	5.168E-119	3.992E + 00	0.000E + 00	1.498E-84	2.306E-01	5.893E + 00
F5	Mean	2.841E + 01	2.811E + 01	2.443E + 01	1.027E + 02	**1.615E + 00**	2.640E + 01	8.920E + 01	2.420E + 04
	STD	4.178E-01	5.721E-01	1.044E + 01	5.954E + 01	**6.144E + 00**	2.677E-01	7.287E + 01	5.148E + 04
F6	Mean	4.291E + 00	3.307E + 00	2.502E-04	9.873E-03	**1.893E-07**	2.198E-07	2.659E-04	5.501E + 00
	STD	4.695E-01	6.626E-01	3.208E-04	6.021E-03	**3.245E-07**	1.157E-07	4.051E-04	2.084E + 01
F7	Mean	1.827E-04	1.165E-04	4.554E-04	1.554E-01	**8.907E-05**	1.702E-03	1.791E-01	8.725E-02
	STD	1.797E-04	1.265E-04	3.847E-04	6.495E-02	**7.564E-05**	6.526E-04	6.943E-02	4.202E-02
Multimodal Functions (F8–F13)									
F8	Mean	−2.032E + 04	−6.151E + 03	−7.340E + 03	−8.150E + 03	−1.257E + 04	−6.007E + 03	−4.620E + 03	−8.024E + 03
	STD	4.236E + 04	4.273E + 02	8.393E + 02	6.457E + 02	2.742E-04	1.674E + 03	1.359E + 03	1.269E + 03
F9	Mean	**0.000E + 00**	1.473E-11	**0.000E + 00**	6.378E + 01	**0.000E + 00**	2.272E + 01	5.357E + 01	1.339E + 02
	STD	**0.000E + 00**	6.585E-11	**0.000E + 00**	1.807E + 01	**0.000E + 00**	6.146E + 01	1.087E + 01	4.038E + 01

Table 4 continued

Function	Stats	HGSO	SHO	HLOA	FDA	GTO	GMO	PSO	DE
F10	Mean	1.066E-15	4.441E-15	8.882E-16	4.576E + 00	**4.441E-16**	4.263E-15	1.881E-01	2.006E + 00
	STD	7.944E-16	0.000E + 00	0.000E + 00	1.095E + 00	**0.000E + 00**	7.944E-16	3.839E-01	1.184E + 00
F11	Mean	**0.000E + 00**	**0.000E + 00**	**0.000E + 00**	2.127E-01	**0.000E + 00**	**0.000E + 00**	8.467E-03	2.131E-01
	STD	**0.000E + 00**	**0.000E + 00**	**0.000E + 00**	1.709E-01	**0.000E + 00**	**0.000E + 00**	1.119E-02	3.858E-01
F12	Mean	4.030E-01	2.531E-01	2.085E-01	3.374E + 00	2.062E-08	5.654E-03	3.466E-03	2.183E + 04
	STD	8.499E-02	6.937E-02	7.252E-02	1.884E + 00	3.116E-08	2.526E-02	1.893E-02	7.540E + 04
F13	Mean	2.755E + 00	2.047E + 00	1.617E + 00	7.321E + 00	2.134E-03	**1.105E-03**	4.508E-03	3.381E + 04
	STD	1.329E-01	2.625E-01	1.885E-01	9.208E + 00	5.839E-03	**3.381E-03**	5.598E-03	6.787E + 04
Fixed-Dimension Multimodal Functions (F14–F23)									
F14	Mean	1.935E + 00	3.646E + 00	6.043E + 00	1.147E + 00	9.980E-01	5.904E + 00	3.197E + 00	1.130E + 00
	STD	7.864E-01	3.749E + 00	4.059E + 00	4.857E-01	5.831E-17	4.158E + 00	2.852E + 00	4.310E-01
F15	Mean	3.667E-04	3.375E-03	1.094E-02	8.632E-04	5.517E-04	3.639E-04	8.876E-04	2.021E-03
	STD	3.818E-05	7.379E-03	1.561E-02	4.349E-04	4.119E-04	9.225E-05	1.592E-04	5.017E-03
F16	Mean	**−1.032E + 00**	**−1.032E + 00**	**−1.032E + 00**	**−1.032E + 00**	**−1.032E + 00**	−1.030E + 00	**−1.032E + 00**	**−1.032E + 00**
	STD	6.138E-05	8.467E-09	2.696E-07	**2.161E-16**	6.253E-16	2.829E-03	6.454E-16	6.775E-16
F17	Mean	**3.979E-01**	**3.979E-01**	**3.979E-01**	**3.979E-01**	**3.979E-01**	**3.979E-01**	**3.979E-01**	**3.979E-01**
	STD	1.640E-15	**0.000E + 00**	1.134E-04	**0.000E + 00**	**0.000E + 00**	**0.000E + 00**	**0.000E + 00**	**0.000E + 00**
F18	Mean	**3.000E + 00**	**3.000E + 00**	**3.000E + 00**	**3.000E + 00**	**3.000E + 00**	**3.000E + 00**	**3.000E + 00**	**3.000E + 00**
	STD	1.165E-04	5.612E-09	2.433E-06	**1.003E-15**	1.078E-15	1.873E-15	1.435E-15	2.222E-15

Table 4 continued

Function	Stats	HGSO	SHO	HLOA	FDA	GTO	GMO	PSO	DE
F19	Mean	−3.858E + 00	−3.859E + 00	**−3.863E + 00**	**-3.863E + 00**	**−3.863E + 00**	**−3.863E + 00**	**−3.863E + 00**	**−3.863E + 00**
	STD	2.622E-03	3.710E-03	**2.782E-03**	**2.782E-03**	**2.782E-03**	**2.782E-03**	**2.782E-03**	**2.782E-03**
F20	Mean	−3.052E + 00	−3.040E + 00	−3.253E + 00	−3.299E + 00	−3.259E + 00	−3.286E + 00	−3.270E + 00	−3.243E + 00
	STD	2.683E-01	2.807E-01	6.790E-02	2.390E-02	6.328E-02	3.683E-02	5.992E-02	5.701E-02
F21	Mean	−4.755E + 00	−5.880E + 00	−9.649E + 00	−9.904E + 00	**−1.015E + 01**	−8.381E + 00	−7.554E + 00	−9.653E + 00
	STD	5.398E + 00	4.273E + 00	5.044E−01	2.490E−01	**3.209E−07**	1.772E + 00	3.122E + 00	1.902E + 00
F22	Mean	−4.826E + 00	-5.624E + 00	−7.351E + 00	-9.873E + 00	**-1.040E + 01**	−8.931E + 00	−1.008E + 01	−9.226E + 00
	STD	5.577E + 00	4.779E + 00	3.052E + 00	5.303E-01	1.406E-04	1.472E + 00	1.432E + 00	2.684E + 00
F23	Mean	− 4.983E + 00	− 5.802E + 00	− 7.087E + 00	− 1.031E + 01	**− 1.054E + 01**	− 9.558E + 00	− 1.018E + 01	− 1.006E + 01
	STD	5.553E + 00	4.735E + 00	3.450E + 00	2.235E-01	**1.098E-04**	9.788E-01	1.360E + 00	1.827E + 00
	WIN	**4**	**4**	**5**	**4**	**15**	**5**	**4**	**4**

Bold values represent the best results

optimizers across the 23 classical benchmark functions. Specifically, in the unimodal functions category (F1-F7), EAO attains the optimal mean values on F1 through F4, highlighting its strong exploitation ability. Meanwhile, GTO achieves better results on F5, F6, and F7.

In the multimodal functions category (F8-F13), EAO outperforms other optimizers in F9, F10, F11, and F12, showing its ability to navigate complex landscapes and avoid local optima. For F8 and F13, HGSO and GMO, respectively, provide the best performance. Moreover, within the fixed-dimension multimodal functions (F14-F23), EAO outperforms on most of the benchmark functions, achieving the best mean results in almost all functions and highlighting its good balance between exploration and exploitation, where it secures the highest number of overall wins (18 out of 23).

Table 5 Statistical results of Wilcoxon signed-rank test for EAO versus other compared optimizers over the first dataset (23 benchmark functions)

Function	EAO versus WOA	EAO versus MVO	EAO versus SCA	EAO versus GWO
UM	5/2/0	6/0/1	7/0/0	7/0/0
MM	2/3/1	4/2/0	6/0/0	5/1/0
FDMM	6/2/2	5/4/1	9/0/1	4/4/2
Total	13/7/3	15/6/2	22/0/1	16/5/2

Function	EAO versus WSO	EAO versus HGSO	EAO versus FDA	EAO versus GMO
UM	6/1/0	6/1/0	7/0/0	7/0/0
MM	4/0/2	5/0/1	4/2/0	4/2/0
FDMM	6/2/2	5/2/3	4/4/2	3/5/2
Total	16/3/4	16/3/4	15/6/2	14/7/2

Function	EAO versus GTO	EAO versus SHO	EAO versus HLOA	EAO versus MFO
UM	5/1/1	6/0/1	7/0/0	6/1/0
MM	4/0/2	4/2/0	6/0/0	4/0/2
FDMM	6/3/1	5/4/1	9/1/0	6/2/2
Total[±/=]	15/4/4	15/6/2	22/1/0	16/3/4

Function	EAO versus DE	EAO versus PSO
UM	6/0/1	7/0/0
MM	3/1/2	3/2/1
FDMM	10/0/0	8/0/2
Total[±/=]	19/1/3	18/2/3

Table 6 Comparison results of the IEEE CEC2017 benchmark functions (F1–F30) with Dim=10, Runs=30, and Iterations=1000

Function Stats.	Stats.	EAO	WOA	MFO	MVO	SCA	GWO	WSO	HGSO
F1	Mean	**1.000E + 02**	1.042E + 07	6.376E + 07	8.232E + 03	8.707E + 08	1.489E + 07	7.191E + 06	1.023E + 09
	STD	**3.937E-03**	2.125E + 07	2.531E + 08	3.907E + 03	4.772E + 08	5.233E + 07	2.969E + 07	3.484E + 08
F2	Mean	**2.000E + 02**	5.897E + 06	5.771E + 09	2.573E + 02	5.441E + 07	3.194E + 07	5.350E + 09	1.653E + 08
	STD	**1.515E-07**	1.305E + 07	2.414E + 10	5.137E + 01	1.111E + 08	1.055E + 08	2.282E + 10	3.053E + 08
F3	Mean	**3.000E + 02**	2.625E + 03	7.769E + 03	**3.000E + 02**	2.034E + 03	2.162E + 03	1.357E + 03	2.035E + 03
	STD	1.112E-08	2.143E + 03	8.827E + 03	1.573E-02	1.123E + 03	2.876E + 03	1.574E + 03	9.893E + 02
F4	Mean	**4.000E + 02**	4.366E + 02	4.153E + 02	4.042E + 02	4.701E + 02	4.189E + 02	4.178E + 02	4.711E + 02
	STD	**8.351E-10**	3.464E + 01	2.753E + 01	1.674E + 00	3.361E + 01	2.034E + 01	2.477E + 01	1.888E + 01
F5	Mean	5.310E + 02	5.526E + 02	5.318E + 02	5.198E + 02	5.498E + 02	5.168E + 02	5.250E + 02	5.558E + 02
	STD	4.591E + 00	1.812E + 01	1.185E + 01	8.274E + 00	8.402E + 00	7.826E + 00	1.546E + 01	5.101E + 00
F6	Mean	**6.000E + 02**	6.391E + 02	6.018E + 02	6.017E + 02	6.197E + 02	6.011E + 02	6.049E + 02	6.270E + 02
	STD	**1.545E-05**	1.609E + 01	2.717E + 00	1.641E + 00	4.187E + 00	1.690E + 00	8.589E + 00	5.925E + 00
F7	Mean	7.415E + 02	7.871E + 02	7.429E + 02	7.264E + 02	7.748E + 02	7.294E + 02	7.441E + 02	7.808E + 02
	STD	4.518E + 00	2.267E + 01	1.254E + 01	8.542E + 00	1.181E + 01	9.269E + 00	3.202E + 01	9.853E + 00
F8	Mean	8.323E + 02	8.443E + 02	8.291E + 02	8.208E + 02	8.422E + 02	8.128E + 02	8.203E + 02	8.377E + 02
	STD	4.634E + 00	1.864E + 01	1.313E + 01	1.005E + 01	8.952E + 00	4.647E + 00	1.441E + 01	3.650E + 00
F9	Mean	**9.000E + 02**	1.498E + 03	1.001E + 03	9.002E + 02	1.020E + 03	9.282E + 02	1.068E + 03	1.034E + 03
	STD	**2.926E-09**	2.986E + 02	2.994E + 02	4.311E-01	3.758E + 01	4.731E + 01	3.169E + 02	3.697E + 01
F10	Mean	1.890E + 03	2.073E + 03	2.092E + 03	1.665E + 03	2.320E + 03	1.712E + 03	1.832E + 03	2.640E + 03
	STD	2.816E + 02	3.664E + 02	2.909E + 02	2.727E + 02	2.756E + 02	2.623E + 02	4.291E + 02	1.612E + 02
F11	Mean	**1.106E + 03**	1.221E + 03	1.230E + 03	1.118E + 03	1.228E + 03	1.165E + 03	1.175E + 03	1.340E + 03
	STD	**2.272E + 00**	8.049E + 01	1.285E + 02	1.364E + 01	6.491E + 01	7.985E + 01	1.078E + 02	9.261E + 01

Table 6 continued

Function Stats.	Stats.	EAO	WOA	MFO	MVO	SCA	GWO	WSO	HGSO
F12	Mean	**1.210E + 03**	4.701E + 06	1.754E + 06	4.678E + 05	2.105E + 07	4.815E + 05	2.265E + 06	1.167E + 07
	STD	**1.872E + 01**	4.859E + 06	4.722E + 06	6.724E + 05	1.460E + 07	7.392E + 05	4.518E + 06	6.462E + 06
F13	Mean	**1.311E + 03**	1.633E + 04	9.386E + 03	9.926E + 03	3.640E + 04	1.114E + 04	2.174E + 04	5.161E + 04
	STD	**3.175E + 00**	1.174E + 04	1.041E + 04	1.002E + 04	3.608E + 04	9.028E + 03	2.719E + 04	4.556E + 04
F14	Mean	**1.420E + 03**	2.439E + 03	3.985E + 03	1.540E + 03	2.112E + 03	3.134E + 03	3.179E + 03	3.284E + 03
	STD	**3.303E + 00**	1.353E + 03	3.138E + 03	2.221E + 02	1.242E + 03	1.903E + 03	2.139E + 03	1.292E + 03
F15	Mean	**1.501E + 03**	6.571E + 03	7.153E + 03	1.734E + 03	2.727E + 03	5.232E + 03	5.868E + 03	6.357E + 03
	STD	**6.548E-01**	5.125E + 03	8.969E + 03	3.925E + 02	9.362E + 02	2.823E + 03	5.694E + 03	2.417E + 03
F16	Mean	**1.617E + 03**	1.860E + 03	1.764E + 03	1.733E + 03	1.740E + 03	1.736E + 03	1.791E + 03	1.904E + 03
	STD	**1.787E + 01**	1.187E + 02	1.051E + 02	9.761E + 01	7.174E + 01	8.648E + 01	1.443E + 02	8.228E + 01
F17	Mean	1.756E + 03	1.792E + 03	1.770E + 03	1.777E + 03	1.783E + 03	1.766E + 03	1.768E + 03	1.787E + 03
	STD	8.438E + 00	4.766E + 01	4.409E + 01	4.772E + 01	1.004E + 01	3.601E + 01	3.652E + 01	9.098E + 00
F18	Mean	**1.804E + 03**	1.973E + 04	2.209E + 04	1.804E + 04	1.510E + 05	2.674E + 04	4.758E + 04	4.964E + 05
	STD	**4.953E + 00**	1.199E + 04	1.569E + 04	1.436E + 04	1.034E + 05	1.573E + 04	6.347E + 04	3.398E + 05
F19	Mean	**1.901E + 03**	1.019E + 05	2.485E + 04	2.220E + 03	5.626E + 03	9.231E + 03	9.155E + 03	1.492E + 04
	STD	**8.944E-01**	2.159E + 05	4.822E + 04	5.823E + 02	5.165E + 03	7.078E + 03	6.807E + 03	8.698E + 03
F20	Mean	2.049E + 03	2.215E + 03	2.101E + 03	2.119E + 03	2.093E + 03	2.082E + 03	2.113E + 03	2.156E + 03
	STD	1.362E + 01	5.805E + 01	6.594E + 01	7.885E + 01	2.195E + 01	5.653E + 01	7.565E + 01	3.908E + 01
F21	Mean	**2.213E + 03**	2.333E + 03	2.325E + 03	2.280E + 03	2.267E + 03	2.318E + 03	2.322E + 03	2.292E + 03
	STD	3.920E + 01	5.098E + 01	3.112E + 01	5.353E + 01	6.648E + 01	7.683E + 00	3.330E + 01	4.861E + 01
F22	Mean	**2.277E + 03**	2.380E + 03	2.310E + 03	2.327E + 03	2.371E + 03	2.308E + 03	2.307E + 03	2.412E + 03
	STD	**4.662E + 01**	2.822E + 02	1.713E + 01	1.576E + 02	3.954E + 01	2.347E + 01	4.825E + 00	5.048E + 01

Table 6 continued

Function Stats.	Stats.	EAO	WOA	MFO	MVO	SCA	GWO	WSO	HGSO
F23	Mean	2.631E + 03	2.655E + 03	2.630E + 03	2.621E + 03	2.657E + 03	2.626E + 03	2.627E + 03	2.685E + 03
	STD	5.115E + 00	2.517E + 01	1.050E + 01	1.022E + 01	8.510E + 00	1.119E + 01	1.602E + 01	1.009E + 01
F24	Mean	2.733E + 03	2.767E + 03	2.765E + 03	2.730E + 03	2.785E + 03	2.740E + 03	2.746E + 03	**2.595E + 03**
	STD	7.917E + 01	7.551E + 01	9.439E + 00	6.364E + 01	5.463E + 00	5.684E + 01	6.057E + 01	**6.029E + 01**
F25	Mean	**2.898E + 03**	2.951E + 03	2.949E + 03	2.919E + 03	2.964E + 03	2.931E + 03	2.942E + 03	2.971E + 03
	STD	**1.403E-01**	2.293E + 01	3.409E + 01	3.311E + 01	2.538E + 01	1.715E + 01	2.928E + 01	1.611E + 01
F26	Mean	2.890E + 03	3.505E + 03	3.124E + 03	2.956E + 03	3.100E + 03	3.048E + 03	3.144E + 03	3.302E + 03
	STD	5.477E + 01	5.688E + 02	3.544E + 02	2.698E + 02	4.838E + 01	2.933E + 02	4.171E + 02	1.095E + 02
F27	Mean	**3.089E + 03**	3.136E + 03	3.094E + 03	3.097E + 03	3.103E + 03	3.101E + 03	3.110E + 03	3.138E + 03
	STD	**1.929E-01**	4.278E + 01	3.948E + 00	2.052E + 01	1.852E + 00	1.437E + 01	3.307E + 01	1.684E + 01
F28	Mean	**3.100E + 03**	3.363E + 03	3.352E + 03	3.295E + 03	3.305E + 03	3.373E + 03	3.369E + 03	3.455E + 03
	STD	**6.037E-05**	1.477E + 02	7.925E + 01	1.366E + 02	8.955E + 01	8.153E + 01	1.072E + 02	1.568E + 02
F29	Mean	3.165E + 03	3.364E + 03	3.228E + 03	3.224E + 03	3.247E + 03	3.212E + 03	3.229E + 03	3.268E + 03
	STD	8.080E + 00	1.261E + 02	5.535E + 01	5.978E + 01	4.403E + 01	4.605E + 01	7.206E + 01	2.569E + 01
F30	Mean	**3.400E + 03**	9.496E + 05	7.901E + 05	5.087E + 05	1.397E + 06	5.247E + 05	5.821E + 05	3.153E + 05
	STD	**1.626E + 01**	1.237E + 06	1.048E + 06	6.354E + 05	7.036E + 05	7.015E + 05	6.673E + 05	4.106E + 05
	Win	**20**	**0**	**0**	**1**	**0**	**0**	**0**	**1**

Function	Stats.	SHO	HLOA	FDA	GTO	GMO	PSO	DE
F1	Mean	8.416E + 08	1.788E + 04	1.809E + 03	2.442E + 03	1.228E + 03	1.340E + 03	1.192E + 03
	STD	1.193E + 09	3.329E + 04	1.888E + 03	2.003E + 03	1.383E + 03	1.407E + 03	1.494E + 03
F2	Mean	1.595E + 09	4.904E + 02	2.000E + 02	2.000E + 02	2.389E + 02	**2.000E + 02**	1.467E + 04
	STD	5.124E + 09	6.850E + 02	6.204E-05	7.072E-04	3.290E + 01	3.704E-05	3.569E + 04

Table 6 continued

Function	Stats.	SHO	HLOA	FDA	GTO	GMO	PSO	DE
F3	Mean	4.143E + 03	3.054E + 02	**3.000E + 02**	**3.000E + 02**	**3.000E + 02**	**3.000E + 02**	3.342E + 02
	STD	2.962E + 03	1.173E + 01	9.837E-11	1.389E-11	1.158E-05	**4.088E-14**	1.518E + 02
F4	Mean	4.510E + 02	4.131E + 02	4.008E + 02	4.051E + 02	4.053E + 02	4.051E + 02	4.054E + 02
	STD	4.693E + 01	2.185E + 01	7.639E-01	1.475E + 01	1.064E + 00	1.221E + 01	2.027E + 00
F5	Mean	5.379E + 02	5.769E + 02	5.193E + 02	5.239E + 02	5.199E + 02	5.491E + 02	**5.097E + 02**
	STD	1.290E + 01	2.643E + 01	7.433E + 00	9.522E + 00	1.137E + 01	1.479E + 01	**6.634E + 00**
F6	Mean	6.133E + 02	6.480E + 02	6.001E + 02	6.054E + 02	6.011E + 02	6.109E + 02	**6.000E + 02**
	STD	5.742E + 00	1.443E + 01	3.753E-01	4.440E + 00	2.131E + 00	8.715E + 00	3.743E-05
F7	Mean	7.626E + 02	7.914E + 02	7.371E + 02	7.532E + 02	7.302E + 02	7.279E + 02	**7.241E + 02**
	STD	1.323E + 01	2.108E + 01	1.222E + 01	1.514E + 01	1.569E + 01	7.927E + 00	**9.257E + 00**
F8	Mean	8.260E + 02	8.457E + 02	8.249E + 02	8.263E + 02	8.163E + 02	8.193E + 02	**8.082E + 02**
	STD	7.728E + 00	2.250E + 01	8.238E + 00	7.890E + 00	1.068E + 01	8.026E + 00	**5.113E + 00**
F9	Mean	1.019E + 03	1.618E + 03	9.039E + 02	9.536E + 02	**9.000E + 02**	9.316E + 02	9.001E + 02
	STD	7.456E + 01	1.758E + 02	5.486E + 00	7.393E + 01	1.016E-01	7.878E + 01	2.082E-01
F10	Mean	1.914E + 03	2.407E + 03	1.783E + 03	1.831E + 03	1.704E + 03	1.985E + 03	**1.430E + 03**
	STD	3.819E + 02	2.465E + 02	2.730E + 02	2.857E + 02	2.656E + 02	2.526E + 02	**2.640E + 02**
F11	Mean	1.180E + 03	1.217E + 03	1.111E + 03	1.124E + 03	1.123E + 03	1.141E + 03	1.108E + 03
	STD	8.182E + 01	9.184E + 01	8.964E + 00	1.600E + 01	9.911E + 00	2.386E + 01	8.962E + 00
F12	Mean	8.383E + 05	2.082E + 05	1.160E + 04	1.220E + 04	8.803E + 05	1.212E + 04	1.187E + 04
	STD	5.667E + 05	4.427E + 05	1.260E + 04	1.111E + 04	5.829E + 05	8.917E + 03	1.277E + 04
F13	Mean	8.975E + 03	3.483E + 03	1.607E + 03	1.482E + 03	1.073E + 04	7.611E + 03	1.34 9E + 03
	STD	5.041E + 03	2.637E + 03	2.861E + 02	1.283E + 02	3.917E + 03	6.025E + 03	1.633E + 01
F14	Mean	3.892E + 03	4.330E + 03	1.470E + 03	1.457E + 03	4.078E + 03	2.143E + 03	1.427E + 03
	STD	1.297E + 03	8.357E + 03	3.200E + 01	2.755E + 01	1.037E + 03	9.978E + 02	8.173E + 00

Table 6 continued

Function	Stats.	SHO	HLOA	FDA	GTO	GMO	PSO	DE
F15	Mean	3.097E + 03	3.149E + 03	1.602E + 03	1.585E + 03	1.344E + 04	2.573E + 03	1.506E + 03
	STD	1.104E + 03	2.739E + 03	7.718E + 01	8.197E + 01	5.151E + 03	1.120E + 03	1.214E + 01
F16	Mean	1.797E + 03	2.070E + 03	1.670E + 03	1.689E + 03	1.909E + 03	1.900E + 03	1.619E + 03
	STD	1.430E + 02	1.921E + 02	6.042E + 01	1.009E + 02	1.466E + 02	1.254E + 02	2.663E + 01
F17	Mean	1.755E + 03	1.927E + 03	1.745E + 03	1.748E + 03	1.772E + 03	1.756E + 03	**1.716E + 03**
	STD	2.004E + 01	1.267E + 02	2.393E + 01	1.871E + 01	1.623E + 01	3.427E + 01	**1.849E + 01**
F18	Mean	1.993E + 04	1.164E + 04	2.070E + 03	2.143E + 03	9.085E + 03	1.282E + 04	2.801E + 03
	STD	1.015E + 04	1.486E + 04	2.572E + 02	4.727E + 02	4.412E + 03	9.793E + 03	5.330E + 03
F19	Mean	6.792E + 03	3.228E + 03	1.946E + 03	1.930E + 03	7.152E + 03	4.763E + 03	**1.901E + 03**
	STD	4.429E + 03	4.039E + 03	3.333E + 01	3.678E + 01	2.591E + 03	4.173E + 03	1.887E + 00
F20	Mean	2.074E + 03	2.277E + 03	2.047E + 03	2.056E + 03	2.146E + 03	2.119E + 03	**2.008E + 03**
	STD	5.345E + 01	9.481E + 01	1.191E + 01	3.048E + 01	5.625E + 01	6.677E + 01	**2.255E + 01**
F21	Mean	2.313E + 03	2.347E + 03	2.268E + 03	**2.213E + 03**	2.298E + 03	2.311E + 03	2.299E + 03
	STD	4.240E + 01	5.425E + 01	6.187E + 01	**2.625E + 01**	4.341E + 01	5.750E + 01	3.333E + 01
F22	Mean	2.380E + 03	2.514E + 03	2.300E + 03	2.298E + 03	2.299E + 03	2.486E + 03	2.300E + 03
	STD	5.580E + 01	5.153E + 02	1.469E + 01	2.096E + 01	1.471E + 01	4.568E + 02	3.289E-01
F23	Mean	2.655E + 03	2.720E + 03	2.619E + 03	2.623E + 03	2.616E + 03	2.698E + 03	**2.610E + 03**
	STD	1.446E + 01	5.234E + 01	8.248E + 00	1.099E + 01	1.244E + 01	3.495E + 01	**6.903E + 00**
F24	Mean	2.761E + 03	2.836E + 03	2.745E + 03	2.692E + 03	2.673E + 03	2.763E + 03	2.739E + 03
	STD	6.299E + 01	4.199E + 01	5.874E + 01	1.140E + 02	1.069E + 02	1.379E + 02	4.280E + 00
F25	Mean	2.958E + 03	2.936E + 03	2.932E + 03	2.930E + 03	2.937E + 03	2.925E + 03	2.919E + 03
	STD	9.723E + 01	4.012E + 01	2.254E + 01	2.385E + 01	1.681E + 01	2.803E + 01	2.367E + 01
F26	Mean	3.137E + 03	3.732E + 03	2.935E + 03	2.938E + 03	**2.845E + 03**	3.165E + 03	2.955E + 03
	STD	2.435E + 02	4.845E + 02	7.643E + 01	1.141E + 02	**7.592E + 01**	3.764E + 02	1.648E + 02

Table 6 continued

Function	Stats.	SHO	HLOA	FDA	GTO	GMO	PSO	DE
F27	Mean	3.129E + 03	3.232E + 03	3.093E + 03	3.096E + 03	3.095E + 03	3.166E + 03	3.094E + 03
	STD	2.164E + 01	8.363E + 01	3.218E + 00	7.079E + 00	2.547E + 00	5.448E + 01	3.489E + 00
F28	Mean	3.415E + 03	3.407E + 03	3.243E + 03	3.268E + 03	3.299E + 03	3.211E + 03	3.281E + 03
	STD	1.764E + 02	2.118E + 02	1.452E + 02	1.730E + 02	1.334E + 02	5.896E + 01	1.266E + 02
F29	Mean	3.263E + 03	3.459E + 03	3.201E + 03	3.212E + 03	3.206E + 03	3.240E + 03	**3.140E + 03**
	STD	5.276E + 01	1.835E + 02	5.433E + 01	5.267E + 01	3.190E + 01	4.462E + 01	**1.173E + 01**
F30	Mean	6.753E + 05	1.740E + 06	1.819E + 05	1.831E + 06	3.375E + 05	1.168E + 04	1.886E + 05
	STD	7.449E + 05	3.554E + 06	3.776E + 05	2.372E + 06	4.544E + 05	6.634E + 03	3.721E + 05
	Win	**0**	**0**	**1**	**2**	**3**	3	10

Bold values represent the best results

Following EAO, GTO ranks second with 15 wins, and WSO achieves 7 wins. WOA, GMO, and HLOA each secure 5 wins, while MFO, GWO, HGSO, SHO, FDA, PSO, and DE follow with 4 wins each. Lastly, MVO and SCA obtain 2 wins.

To evaluate the significance of EAO's performance, we conducted a Wilcoxon signed-rank test on the 23 classical benchmark functions at a significance level of $\alpha = 0.05$. As shown in Table 5, EAO consistently outperforms most compared optimizers, particularly in unimodal functions (UM), where it achieves clear wins over algorithms such as SCA, GWO, FDA, GMO, and HLOA.

In the multimodal (MM) category, EAO also demonstrates superior results against the majority of competitors, including WSO, while exhibiting comparable performance to MVO and FDA on selected functions. Moreover, EAO maintains its strong edge in fixed-dimension multimodal (FDMM) functions, where it obtains notably high win counts, especially against PSO, DE, SCA, and HLOA, with 18, 19, 22, and 22 wins, respectively. For a comprehensive breakdown of the p-values for each function and optimizer, refer to Table 19 in Appendix A.

5.2 Performance analysis on IEEE CEC2017 benchmark functions (F1-F30)

EAO exhibits outstanding performance compared to other state-of-the-art optimizers on the IEEE CEC2017 benchmark functions with dimensionality ($d = 10$). As presented in Table 6, EAO achieves 20 wins out of 30 benchmark functions, demonstrating its superior ability to tackle both simple and complex optimization landscapes. The CEC2017 benchmark includes a diverse set of functions characterized by features such as multimodality, non-separability, and compositional structures, which are particularly challenging for optimization algorithms.

EAO's consistently superior results, especially on the more difficult functions, illustrate its robustness and effectiveness in navigating complex search spaces. Meanwhile, competitive optimizers such as DE, GMO, PSO, and GTO achieve 10, 3, 3, and 2 wins, respectively.

Table 7 presents the results of the Wilcoxon signed-rank test, showing that EAO significantly outperforms most competitors in all function categories. In the unimodal (UN) category, EAO attains clear wins over WOA, MFO, MVO, SCA, GWO, HGSO, and SHO. For the multimodal (MM) functions, EAO also demonstrates significantly better results compared to most optimizers. In the hybrid (HF) and composition (CF) functions, EAO continues to show strong performance against all other algorithms, significantly outperforming all compared optimizers such as DE, WOA, MFO, PSO, MVO, SCA, GWO, HGSO, SHO, GTO, and GMO. For a comprehensive breakdown of the p-values for each function and optimizer, refer to Table 20 in Appendix A.

Moreover, EAO exhibits superior performance compared to other state-of-the-art optimizers on the IEEE CEC2017 benchmark functions with high dimensionality ($d = 100$). As shown in Table 8, EAO achieves 16 wins out of 30 benchmark functions. GMO and DE each secure 4 wins, while WSO and PSO follow with 2 wins, highlighting EAO's strong performance relative to both traditional methods and other recent and competitive optimizers.

Table 7 Statistical results of Wilcoxon signed-rank test for EAO versus other compared optimizers over the IEEE CEC2017 benchmark dataset

Function	EAO versus WOA	EAO versus SCA	EAO versus MFO	EAO versus MVO
UN	3/0/0	3/0/0	3/0/0	2/0/1
MM	7/0/0	7/0/0	6/0/1	4/1/2
HF	10/0/0	10/0/0	9/0/1	9/0/1
CF	10/0/0	10/0/0	10/0/0	9/0/1
Total	30/0/0	30/0/0	28/0/2	24/1/5

Function	EAO versus GWO	EAO versus WSO	EAO versus HGSO	EAO versus SHO
UN	3/0/0	2/1/0	3/0/0	3/0/0
MM	4/1/2	1/4/2	7/0/0	6/0/1
HF	10/0/0	5/5/0	10/0/0	10/0/0
CF	10/0/0	8/1/1	9/1/0	10/0/0
Total	27/1/2	16/11/3	29/1/0	29/0/1

Function	EAO versus HLOA	EAO versus FDA	EAO versus GTO	EAO versus GMO
UN	2/1/0	0/2/1	1/2/0	2/1/0
MM	7/0/0	3/2/2	5/1/1	2/1/4
HF	10/0/0	8/2/0	9/0/1	10/0/0
CF	10/0/0	10/0/0	8/0/2	7/1/2
Total	29/1/0	21/6/3	23/3/4	21/3/6

Function	EAO versus DE	EAO versus PSO
UN	2/1/0	3/0/0
MM	4/2/1	4/1/2
HF	6/1/3	8/1/1
CF	6/0/4	8/0/2
Total[±/=]	18/4/8	23/2/5

The CEC2017 benchmark functions at ($d = 100$) are characterized by increased complexity, including high-dimensional multimodality, non-separability, and composite landscapes, all of which pose substantial challenges for optimization algorithms. EAO's superior performance in this high-dimensional setting highlights its robust capability to navigate and exploit complex search spaces effectively, outperforming many optimizers that struggle to maintain efficiency as the dimensionality increases.

5.3 Results of the IEEE CEC2022 benchmark functions (F1-F12)

EAO continues to exhibit strong performance on the IEEE CEC2022 benchmark functions across different dimensionalities ($d = 10$ and $d = 20$). As shown in

Table 8 Comparison results of the IEEE CEC2017 benchmark functions (F1–F30) with Dim=100, Runs=30, and Iterations=10 000

Function	Statistics	EAO	WOA	MFO	MVO	SCA	GWO	WSO	HGSO
F1	Mean	9.496E + 03	3.649E + 07	1.284E + 11	2.430E + 05	1.515E + 11	3.055E + 10	4.288E + 09	1.479E + 11
	Std	1.234E + 04	1.940E + 07	4.702E + 10	3.388E + 04	9.846E + 09	7.873E + 09	3.288E + 09	2.437E + 10
F2	Mean	3.155E + 13	2.890E + 133	1.389E + 150	2.046E + 42	1.537E + 148	3.875E + 137	4.090E + 68	3.488E + 146
	Std	8.714E + 13	9.106E + 133	2.892E + 150	6.437E + 42	4.224E + 148	1.225E + 138	1.108E + 69	1.072E + 147
F3	Mean	**3.000E + 02**	5.826E + 05	4.761E + 05	4.621E + 02	2.868E + 05	1.978E + 05	5.775E + 04	2.830E + 05
	Std	**6.218E-08**	1.536E + 05	1.956E + 05	5.386E + 01	2.033E + 04	2.296E + 04	1.047E + 04	1.598E + 04
F4	Mean	**5.502E + 02**	9.762E + 02	1.955E + 04	6.424E + 02	2.649E + 04	2.837E + 03	1.065E + 03	2.854E + 04
	Std	**4.675E + 01**	1.195E + 02	1.164E + 04	2.965E + 01	2.133E + 03	4.911E + 02	2.984E + 02	4.952E + 03
F5	Mean	**6.569E + 02**	1.410E + 03	1.673E + 03	9.415E + 02	1.857E + 03	1.068E + 03	1.084E + 03	1.824E + 03
	Std	**2.479E + 01**	6.865E + 01	1.546E + 02	7.238E + 01	3.950E + 01	1.041E + 02	5.761E + 01	5.093E + 01
F6	Mean	6.003E + 02	6.833E + 02	6.668E + 02	6.412E + 02	6.893E + 02	6.259E + 02	6.317E + 02	6.946E + 02
	Std	1.812E-01	1.205E + 01	9.004E + 00	6.853E + 00	3.914E + 00	3.431E + 00	3.875E + 00	2.429E + 00
F7	Mean	**1.184E + 03**	3.262E + 03	4.352E + 03	1.427E + 03	3.326E + 03	1.767E + 03	2.452E + 03	3.279E + 03
	Std	**2.790E + 02**	1.193E + 02	5.948E + 02	1.004E + 02	1.390E + 02	1.715E + 02	1.300E + 02	9.540E + 01
F8	Mean	**9.779E + 02**	1.840E + 03	2.019E + 03	1.286E + 03	2.198E + 03	1.364E + 03	1.459E + 03	2.237E + 03
	Std	**3.532E + 01**	9.113E + 01	1.499E + 02	1.104E + 02	5.210E + 01	1.883E + 02	4.661E + 01	6.477E + 01
F9	Mean	1.087E + 03	3.854E + 04	4.404E + 04	2.750E + 04	6.338E + 04	2.228E + 04	5.055E + 04	6.538E + 04
	Std	6.943E + 01	6.497E + 03	9.460E + 03	1.104E + 04	5.420E + 03	1.177E + 04	4.067E + 03	3.184E + 03
F10	Mean	3.097E + 04	2.111E + 04	1.641E + 04	1.337E + 04	3.106E + 04	1.355E + 04	**1.177E + 04**	2.659E + 04
	Std	5.445E + 02	2.146E + 03	1.132E + 03	1.064E + 03	9.480E + 02	1.255E + 03	**1.077E + 03**	8.140E + 02
F11	Mean	**1.746E + 03**	6.687E + 03	1.330E + 05	2.560E + 03	6.551E + 04	4.208E + 04	3.863E + 03	1.275E + 05
	Std	**1.408E + 02**	1.457E + 03	8.815E + 04	2.379E + 02	1.185E + 04	9.125E + 03	3.408E + 03	1.420E + 04
F12	Mean	**1.150E + 05**	6.410E + 08	2.959E + 10	1.151E + 08	5.259E + 10	4.576E + 09	2.166E + 07	6.654E + 10
	Std	**3.134E + 04**	2.037E + 08	1.295E + 10	5.102E + 07	6.387E + 09	3.339E + 09	3.614E + 07	1.097E + 10

Table 8 continued

Function	Statistics	EAO	WOA	MFO	MVO	SCA	GWO	WSO	HGSO
F13	Mean	**5.007E + 03**	1.013E + 05	4.896E + 09	9.616E + 04	8.126E + 09	5.059E + 08	7.056E + 03	9.855E + 09
	Std	**2.167E + 03**	5.454E + 04	3.719E + 09	3.490E + 04	2.533E + 09	5.316E + 08	2.334E + 03	2.734E + 09
F14	Mean	**1.515E + 03**	1.948E + 06	1.260E + 07	6.334E + 05	2.086E + 07	2.540E + 06	5.888E + 04	1.580E + 07
	Std	**2.138E + 01**	9.584E + 05	2.590E + 07	2.707E + 05	7.792E + 06	1.544E + 06	3.593E + 04	2.630E + 06
F15	Mean	**1.913E + 03**	1.187E + 05	5.620E + 08	1.017E + 05	2.352E + 09	1.522E + 08	5.492E + 03	2.427E + 09
	Std	**6.785E + 01**	2.140E + 05	5.341E + 08	2.042E + 04	5.266E + 08	1.623E + 08	3.691E + 03	8.320E + 08
F16	Mean	7.213E + 03	9.634E + 03	7.620E + 03	4.922E + 03	1.222E + 04	5.372E + 03	5.068E + 03	1.208E + 04
	Std	2.637E + 03	1.153E + 03	5.909E + 02	6.389E + 02	6.166E + 02	4.528E + 02	6.025E + 02	7.625E + 02
F17	Mean	6.527E + 03	6.564E + 03	1.139E + 04	4.293E + 03	1.182E + 04	4.679E + 03	5.192E + 03	1.952E + 04
	Std	2.878E + 02	6.632E + 02	1.158E + 04	4.341E + 02	3.266E + 03	6.048E + 02	6.817E + 02	1.351E + 04
F18	Mean	**2.425E + 03**	2.277E + 06	7.194E + 06	1.052E + 06	2.704E + 07	4.602E + 06	8.603E + 04	2.308E + 07
	Std	**2.107E + 02**	8.885E + 05	8.571E + 06	5.525E + 05	7.470E + 06	4.997E + 06	2.354E + 04	4.662E + 06
F19	Mean	**2.145E + 03**	1.685E + 07	8.016E + 08	5.228E + 06	2.227E + 09	7.995E + 07	4.360E + 03	3.000E + 09
	Std	**4.516E + 01**	4.804E + 06	8.431E + 08	1.874E + 06	5.939E + 08	6.166E + 07	2.264E + 03	6.658E + 08
F20	Mean	7.143E + 03	6.006E + 03	5.730E + 03	4.566E + 03	7.168E + 03	4.013E + 03	**3.722E + 03**	6.576E + 03
	Std	1.759E + 02	5.175E + 02	6.107E + 02	6.291E + 02	3.214E + 02	3.691E + 02	**2.860E + 02**	2.391E + 02
F21	Mean	**2.526E + 03**	3.974E + 03	3.545E + 03	2.794E + 03	3.873E + 03	2.868E + 03	3.219E + 03	4.093E + 03
	Std	**3.051E + 01**	1.103E + 02	6.422E + 01	6.088E + 01	6.123E + 01	5.243E + 01	1.389E + 02	1.578E + 02
F22	Mean	3.337E + 04	2.471E + 04	2.085E + 04	**1.442E + 04**	3.371E + 04	1.729E + 04	1.898E + 04	3.059E + 04
	Std	4.945E + 02	2.299E + 03	1.611E + 03	**1.711E + 03**	4.734E + 02	2.435E + 03	2.351E + 03	9.873E + 02
F23	Mean	3.047E + 03	4.805E + 03	3.693E + 03	3.320E + 03	4.866E + 03	3.381E + 03	4.002E + 03	6.155E + 03
	Std	3.664E + 01	2.373E + 02	8.182E + 01	8.155E + 01	8.464E + 01	5.454E + 01	2.069E + 02	8.872E + 02
F24	Mean	**3.489E + 03**	6.040E + 03	4.267E + 03	3.704E + 03	6.355E + 03	3.948E + 03	5.168E + 03	7.016E + 03
	Std	**4.783E + 01**	3.281E + 02	1.002E + 02	4.764E + 01	1.484E + 02	9.059E + 01	2.138E + 02	2.801E + 02

Table 8 continued

Function	Statistics	EAO	WOA	MFO	MVO	SCA	GWO	WSO	HGSO
F25	Mean	**3.196E + 03**	3.589E + 03	1.214E + 04	3.224E + 03	1.421E + 04	5.104E + 03	3.695E + 03	1.385E + 04
	Std	**6.468E + 01**	8.410E + 01	4.686E + 03	7.828E + 01	1.068E + 03	3.403E + 02	1.787E + 02	1.845E + 03
F26	Mean	7.939E + 03	3.126E + 04	1.709E + 04	1.054E + 04	3.156E + 04	1.233E + 04	1.907E + 04	3.477E + 04
	Std	4.036E + 02	5.027E + 03	1.026E + 03	6.394E + 02	1.949E + 03	1.144E + 03	7.725E + 03	3.169E + 03
F27	Mean	3.507E + 03	5.157E + 03	3.930E + 03	3.375E + 03	6.925E + 03	3.857E + 03	4.758E + 03	**3.200E + 03**
	Std	8.444E + 01	8.432E + 02	1.939E + 02	3.458E + 01	2.303E + 02	1.004E + 02	3.022E + 02	**1.497E-04**
F28	Mean	7.202E + 03	3.751E + 03	1.773E + 04	3.389E + 03	1.959E + 04	6.373E + 03	3.997E + 03	1.908E + 04
	Std	6.235E + 03	5.106E + 01	1.628E + 03	2.120E + 01	2.551E + 03	8.013E + 02	2.683E + 02	2.388E + 03
F29	Mean	**5.789E + 03**	1.394E + 04	9.494E + 03	6.758E + 03	1.452E + 04	7.448E + 03	7.216E + 03	1.655E + 04
	Std	**1.459E + 03**	1.891E + 03	2.284E + 03	5.153E + 02	9.543E + 02	3.733E + 02	7.416E + 02	2.275E + 03
F30	Mean	7.541E + 03	1.778E + 08	1.031E + 09	2.955E + 07	5.993E + 09	5.145E + 08	2.473E + 05	8.520E + 09
	Std	1.884E + 03	7.236E + 07	9.971E + 08	1.021E + 07	1.366E + 09	3.559E + 08	3.206E + 05	2.663E + 09
	Win	**16**	**0**	**0**	**1**	**0**	**0**	**2**	**1**

Function	Statistics	SHO	HLOA	FDA	GTO	GMO	PSO	DE
F1	Mean	1.720E + 11	1.223E + 07	7.951E + 03	6.708E + 03	1.211E + 04	3.266E + 03	**2.059E + 03**
	Std	1.192E + 10	1.078E + 07	1.001E + 04	8.500E + 03	1.051E + 04	3.957E + 03	**1.939E + 03**
F2	Mean	6.973E + 150	4.187E + 70	1.333E + 44	1.996E + 27	**1.510E + 03**	1.138E + 40	1.676E + 48
	Std	2.205E + 151	1.312E + 71	4.183E + 44	6.308E + 27	**3.057E + 03**	1.218E + 40	2.539E + 48
F3	Mean	2.739E + 05	3.238E + 03	3.001E + 02	4.038E + 02	2.206E + 04	1.755E + 04	4.018E + 05
	Std	1.488E + 04	9.114E + 02	1.336E-01	1.692E + 02	4.429E + 03	7.383E + 03	5.667E + 04
F4	Mean	2.766E + 04	7.375E + 02	5.749E + 02	6.485E + 02	6.546E + 02	5.551E + 02	6.152E + 02
	Std	4.371E + 03	6.670E + 01	3.696E + 01	4.315E + 01	2.464E + 01	5.937E + 01	2.846E + 01

Table 8 continued

Function	Statistics	SHO	HLOA	FDA	GTO	GMO	PSO	DE
F5	Mean	1.641E + 03	1.416E + 03	1.330E + 03	1.318E + 03	8.151E + 02	1.202E + 03	1.316E + 03
	Std	4.666E + 01	6.282E + 01	1.058E + 02	5.175E + 01	6.153E + 01	4.032E + 01	1.764E + 01
F6	Mean	6.631E + 02	6.683E + 02	6.544E + 02	6.585E + 02	6.099E + 02	6.501E + 02	**6.000E + 02**
	Std	3.392E + 00	2.469E + 00	3.137E + 00	3.499E + 00	5.589E + 00	1.709E + 00	**3.027E-03**
F7	Mean	2.992E + 03	3.326E + 03	2.598E + 03	2.934E + 03	1.193E + 03	1.217E + 03	1.624E + 03
	Std	1.789E + 02	1.033E + 02	3.325E + 02	2.059E + 02	1.102E + 02	8.001E + 01	3.756E + 01
F8	Mean	1.980E + 03	1.834E + 03	1.686E + 03	1.722E + 03	1.096E + 03	1.503E + 03	1.620E + 03
	Std	7.824E + 01	7.426E + 01	1.383E + 02	1.071E + 02	7.009E + 01	8.092E + 01	1.490E + 01
F9	Mean	3.396E + 04	2.904E + 04	2.422E + 04	2.160E + 04	2.951E + 03	1.837E + 04	**9.035E + 02**
	Std	3.098E + 03	2.104E + 03	3.433E + 03	1.188E + 03	3.055E + 03	1.855E + 03	**2.369E + 00**
F10	Mean	2.042E + 04	2.073E + 04	1.681E + 04	1.630E + 04	1.183E + 04	1.328E + 04	3.066E + 04
	Std	1.188E + 03	2.835E + 03	8.589E + 02	2.082E + 03	1.376E + 03	1.186E + 03	3.902E + 02
F11	Mean	7.241E + 04	6.789E + 03	2.158E + 03	1.991E + 03	2.044E + 03	1.936E + 03	1.765E + 03
	Std	1.975E + 04	7.006E + 03	1.412E + 02	1.685E + 02	1.252E + 02	1.078E + 02	6.454E + 01
F12	Mean	7.744E + 10	4.995E + 07	7.082E + 05	1.260E + 06	2.455E + 07	2.291E + 06	3.267E + 05
	Std	1.595E + 10	1.536E + 07	5.470E + 05	4.723E + 05	1.108E + 07	9.193E + 05	1.292E + 05
F13	Mean	1.068E + 10	7.038E + 04	5.687E + 03	8.361E + 03	4.876E + 04	5.268E + 03	5.234E + 03
	Std	3.783E + 09	5.248E + 04	3.970E + 03	3.274E + 03	1.340E + 04	8.198E + 02	4.141E + 03
F14	Mean	7.009E + 06	1.382E + 05	7.354E + 03	2.251E + 04	1.508E + 05	3.139E + 05	8.449E + 03
	Std	3.230E + 06	7.686E + 04	6.010E + 03	9.400E + 03	2.930E + 04	1.619E + 05	5.833E + 03
F15	Mean	2.411E + 09	1.120E + 04	2.693E + 03	6.880E + 03	2.850E + 04	3.968E + 03	7.824E + 03
	Std	1.321E + 09	5.786E + 03	9.729E + 02	7.629E + 03	8.716E + 03	2.786E + 03	4.146E + 03
F16	Mean	9.459E + 03	7.392E + 03	5.964E + 03	5.956E + 03	**4.481E + 03**	5.036E + 03	9.189E + 03
	Std	1.114E + 03	1.220E + 03	6.974E + 02	8.069E + 02	**1.522E + 03**	4.942E + 02	1.532E + 02

Table 8 continued

Function	Statistics	SHO	HLOA	FDA	GTO	GMO	PSO	DE
F17	Mean	3.691E + 04	6.353E + 03	5.249E + 03	5.440E + 03	**3.733E + 03**	4.677E + 03	6.213E + 03
	Std	4.553E + 04	6.533E + 02	5.226E + 02	6.297E + 02	**4.479E + 02**	2.945E + 02	3.154E + 02
F18	Mean	6.547E + 06	2.990E + 05	5.679E + 04	6.308E + 04	1.851E + 05	5.586E + 05	1.274E + 05
	Std	3.951E + 06	1.258E + 05	4.509E + 04	1.622E + 04	4.087E + 04	9.558E + 04	9.006E + 04
F19	Mean	1.826E + 09	2.949E + 04	3.524E + 03	7.467E + 03	1.498E + 06	4.423E + 03	1.089E + 04
	Std	1.448E + 09	3.413E + 04	1.737E + 03	4.143E + 03	7.235E + 05	3.063E + 03	1.488E + 04
F20	Mean	5.079E + 03	6.460E + 03	5.463E + 03	5.373E + 03	4.021E + 03	5.206E + 03	5.946E + 03
	Std	4.434E + 02	8.479E + 02	3.328E + 02	3.999E + 02	5.229E + 02	4.104E + 02	1.091E + 03
F21	Mean	3.374E + 03	4.160E + 03	3.169E + 03	3.227E + 03	2.700E + 03	3.397E + 03	3.156E + 03
	Std	1.114E + 02	1.897E + 02	1.224E + 02	1.650E + 02	2.020E + 02	7.001E + 01	2.303E + 01
F22	Mean	2.623E + 04	2.207E + 04	2.015E + 04	2.110E + 04	1.501E + 04	1.599E + 04	3.282E + 04
	Std	9.981E + 02	1.728E + 03	1.157E + 03	2.264E + 03	8.286E + 03	1.212E + 03	4.371E + 02
F23	Mean	4.207E + 03	5.094E + 03	3.627E + 03	4.034E + 03	3.158E + 03	4.632E + 03	**2.899E + 03**
	Std	1.067E + 02	4.027E + 02	1.522E + 02	3.134E + 02	5.829E + 01	3.174E + 02	**1.401E + 01**
F24	Mean	6.005E + 03	7.435E + 03	4.234E + 03	4.697E + 03	3.565E + 03	4.884E + 03	4.077E + 03
	Std	4.224E + 02	6.842E + 02	1.818E + 02	3.034E + 02	9.157E + 01	3.063E + 02	5.919E + 01
F25	Mean	1.443E + 04	3.421E + 03	3.248E + 03	3.277E + 03	3.281E + 03	3.218E + 03	3.204E + 03
	Std	1.864E + 03	5.950E + 01	9.490E + 01	7.403E + 01	4.915E + 01	4.202E + 01	4.468E + 01
F26	Mean	3.408E + 04	3.715E + 04	2.050E + 04	2.279E + 04	**4.566E + 03**	1.151E + 04	1.210E + 04
	Std	2.330E + 03	5.154E + 03	3.086E + 03	2.440E + 03	**2.710E + 03**	7.956E + 03	3.301E + 03
F27	Mean	5.484E + 03	4.648E + 03	3.740E + 03	3.875E + 03	3.437E + 03	3.309E + 03	3.327E + 03
	Std	5.788E + 02	3.988E + 02	1.607E + 02	1.821E + 02	5.373E + 01	8.405E + 01	7.331E + 00
F28	Mean	1.768E + 04	3.500E + 03	3.339E + 03	3.376E + 03	3.386E + 03	**3.321E + 03**	3.335E + 03
	Std	2.080E + 03	5.875E + 01	2.673E + 01	2.797E + 01	2.387E + 01	**2.690E + 01**	2.562E + 01

Table 8 continued

Function	Statistics	SHO	HLOA	FDA	GTO	GMO	PSO	DE
F29	Mean	1.274E + 04	1.016E + 04	7.339E + 03	7.433E + 03	6.112E + 03	6.586E + 03	7.974E + 03
	Std	2.658E + 03	1.071E + 03	4.494E + 02	8.009E + 02	3.021E + 02	4.902E + 02	2.080E + 02
F30	Mean	9.135E + 09	1.219E + 06	7.716E + 03	1.219E + 04	9.562E + 06	**4.875E + 03**	8.231E + 03
	Std	3.835E + 09	1.279E + 06	1.381E + 03	4.546E + 03	4.088E + 06	**1.573E + 03**	3.892E + 03
	Win	**0**	**0**	**0**	**0**	**4**	**2**	**4**

Bold values represent the best results

Table 9 Comparison results of the IEEE CEC2022 benchmark functions (F1–F12) with Dim=10, Runs=30, and Iterations=10,000

Function	Stats	EAO	WOA	MFO	MVO	SCA	GWO	WSO	HGSO
F1	Mean	**3.000E + 02**	1.521E + 03	5.464E + 03	**3.000E + 02**	5.825E + 02	1.148E + 03	**3.000E + 02**	8.909E + 02
	STD	**0.000E + 00**	1.006E + 03	8.232E + 03	**5.607E-05**	9.848E + 01	2.471E + 03	5.684E-14	2.515E + 02
F2	Mean	4.004E + 02	4.125E + 02	4.083E + 02	4.066E + 02	4.279E + 02	4.157E + 02	4.013E + 02	4.593E + 02
	STD	1.253E + 00	1.979E + 01	8.312E-01	3.146E + 00	7.054E + 00	2.143E + 01	2.958E + 00	1.686E + 01
F3	Mean	**6.000E + 02**	6.199E + 02	6.031E + 02	6.004E + 02	6.129E + 02	6.002E + 02	6.001E + 02	6.185E + 02
	STD	2.653E-07	7.958E + 00	4.118E + 00	7.722E-01	2.526E + 00	3.027E-01	3.029E-01	2.392E + 00
F4	Mean	**8.046E + 02**	8.340E + 02	8.314E + 02	8.153E + 02	8.275E + 02	8.106E + 02	8.048E + 02	8.267E + 02
	STD	**1.602E + 00**	1.752E + 01	9.969E + 00	6.686E + 00	3.330E + 00	6.604E + 00	1.678E + 00	3.919E + 00
F5	Mean	**9.000E + 02**	1.409E + 03	9.585E + 02	**9.000E + 02**	9.383E + 02	9.055E + 02	9.008E + 02	9.445E + 02
	STD	**0.000E + 00**	4.154E + 02	9.377E + 01	1.108E-05	1.326E + 01	1.101E + 01	1.515E + 00	1.855E + 01
F6	Mean	**1.800E + 03**	3.166E + 03	5.297E + 03	5.422E + 03	5.801E + 03	5.772E + 03	1.805E + 03	4.533E + 05
	STD	**4.885E-02**	1.719E + 03	2.225E + 03	2.483E + 03	1.928E + 05	2.288E + 03	3.801E + 00	3.121E + 05
F7	Mean	2.001E + 03	2.054E + 03	2.028E + 03	2.019E + 03	2.041E + 03	2.023E + 03	2.009E + 03	2.052E + 03
	STD	8.266E-01	2.669E + 01	1.154E + 01	5.837E + 00	3.341E + 00	3.707E + 00	1.020E + 01	6.547E + 00
F8	Mean	2.201E + 03	2.228E + 03	2.226E + 03	2.219E + 03	2.228E + 03	2.221E + 03	2.210E + 03	2.228E + 03
	STD	5.594E-01	3.885E + 00	4.529E + 00	5.916E + 00	2.286E + 00	7.605E + 00	1.136E + 01	2.041E + 00
F9	Mean	2.529E + 03	2.529E + 03	2.530E + 03	2.529E + 03	2.541E + 03	2.555E + 03	2.529E + 03	2.548E + 03
	STD	0.000E + 00	7.781E-03	2.871E-01	2.024E-04	6.521E + 00	2.564E + 01	0.000E + 00	9.948E + 00
F10	Mean	**2.500E + 03**	2.548E + 03	2.501E + 03	2.534E + 03	2.501E + 03	2.535E + 03	2.533E + 03	2.501E + 03
	STD	**4.005E-02**	6.535E + 01	7.569E-01	5.498E + 01	2.033E-01	5.628E + 01	5.294E + 01	5.005E-01
F11	Mean	**2.600E + 03**	2.807E + 03	2.749E + 03	2.630E + 03	2.751E + 03	2.722E + 03	2.630E + 03	2.764E + 03
	STD	**0.000E + 00**	2.079E + 02	5.393E + 01	9.488E + 01	1.605E + 01	1.254E + 02	6.358E + 01	6.882E + 00
F12	Mean	**2.859E + 03**	2.891E + 03	2.863E + 03	2.860E + 03	2.866E + 03	2.863E + 03	2.869E + 03	2.899E + 03
	STD	**2.133E-01**	5.087E + 01	2.273E + 00	1.177E + 00	9.694E-01	2.198E + 00	6.439E + 00	3.687E + 00

Table 9 continued

Function	Stats	EAO	WOA	MFO	MVO	SCA	GWO	WSO	HGSO
	Win	**8**	**0**	**0**	**2**	**0**	**0**	**1**	**0**
Function	Stats	SHO	HLOA	FDA	GTO	GMO	PSO	DE	
F1	Mean	1.425E + 03	**3.000E + 02**	**3.000E + 02**	**3.000E + 02**	**3.000E + 02**	**3.000E + 02**	**3.000E + 02**	
	STD	1.308E + 03	5.662E-13	1.895E-14	5.684E-14	5.359E-14	**0.000E + 00**	**0.000E + 00**	
F2	Mean	4.247E + 02	4.192E + 02	4.030E + 02	4.113E + 02	**4.000E + 02**	4.008E + 02	4.061E + 02	
	STD	3.346E + 01	2.675E + 01	3.620E + 00	2.108E + 01	**2.843E-02**	1.510E + 00	2.485E + 00	
F3	Mean	6.063E + 02	6.446E + 02	**6.000E + 02**	6.050E + 02	6.002E + 02	6.017E + 02	**6.000E + 02**	
	STD	5.751E + 00	8.208E + 00	1.140E-02	5.305E + 00	3.395E-01	2.851E + 00	**0.000E + 00**	
F4	Mean	8.192E + 02	8.400E + 02	8.226E + 02	8.216E + 02	8.173E + 02	8.150E + 02	8.066E + 02	
	STD	4.682E + 00	1.515E + 01	8.052E + 00	8.365E + 00	9.710E + 00	7.101E + 00	3.081E + 00	
F5	Mean	1.056E + 03	1.444E + 03	9.003E + 02	9.338E + 02	**9.000E + 02**	**9.000E + 02**	**9.000E + 02**	
	STD	1.116E + 02	1.873E + 02	3.942E-01	4.722E + 01	4.325E-02	2.271E-02	**0.000E + 00**	
F6	Mean	4.555E + 03	2.515E + 03	1.844E + 03	1.814E + 03	2.544E + 03	3.618E + 03	**1.800E + 03**	
	STD	1.686E + 03	1.939E + 03	5.066E + 01	1.858E + 01	9.873E + 02	2.061E + 03	1.796E-01	
F7	Mean	2.028E + 03	2.111E + 03	2.017E + 03	2.021E + 03	2.021E + 03	2.018E + 03	**2.000E + 03**	
	STD	1.531E + 01	3.959E + 01	8.580E + 00	1.067E + 01	1.269E + 01	1.243E + 01	**1.583E-01**	
F8	Mean	2.220E + 03	2.304E + 03	2.215E + 03	2.222E + 03	2.247E + 03	2.220E + 03	**2.200E + 03**	
	STD	6.416E + 00	6.086E + 01	9.544E + 00	1.421E + 00	4.983E + 01	3.573E + 00	**1.902E-01**	
F9	Mean	2.575E + 03	2.531E + 03	2.529E + 03	2.529E + 03	2.529E + 03	**2.486E + 03**	2.529E + 03	
	STD	2.912E + 01	6.346E + 00	0.000E + 00	0.000E + 00	8.568E-05	**1.710E-12**	0.000E + 00	
F10	Mean	2.548E + 03	2.545E + 03	**2.500E + 03**	2.512E + 03	2.544E + 03	2.540E + 03	2.504E + 03	
	STD	6.087E + 01	7.150E + 01	7.290E-02	3.797E + 01	5.664E + 01	5.705E + 01	1.903E + 01	
F11	Mean	2.812E + 03	2.807E + 03	2.660E + 03	**2.600E + 03**	2.660E + 03	2.643E + 03	2.605E + 03	
	STD	1.381E + 02	1.946E + 02	7.769E + 01	4.608E-12	1.265E + 02	1.135E + 02	2.746E + 01	

Table 9 continued

Function	Stats	SHO	HLOA	FDA	GTO	GMO	PSO	DE
F12	Mean	2.893E + 03	2.888E + 03	2.863E + 03	2.865E + 03	2.864E + 03	2.867E + 03	2.862E + 03
	STD	2.631E + 01	2.599E + 01	2.002E + 00	1.878E + 00	8.275E-01	2.778E + 01	1.040E + 00
	Win	**0**	**1**	**3**	**2**	**3**	**3**	**6**

Bold values represent the best results

Table 10 Comparison results of the IEEE CEC 2022 benchmark functions (F1–F12) with Dim=20, Runs=30, and Iterations=20,000

Function	Stats	EAO	WOA	MFO	MVO	SCA	GWO	WSO	HGSO
F1	Mean	**3.000E + 02**	3.174E + 02	2.772E + 04	**3.000E + 02**	4.985E + 03	6.210E + 03	**3.000E + 02**	1.611E + 04
	STD	3.276E-14	1.615E + 01	1.335E + 04	6.445E-04	1.091E + 03	3.790E + 03	9.645E-10	1.463E + 03
F2	Mean	4.442E + 02	4.550E + 02	5.032E + 02	4.497E + 02	5.675E + 02	4.715E + 02	4.249E + 02	6.217E + 02
	STD	8.727E + 00	2.035E + 01	4.649E + 01	6.575E + 00	1.808E + 01	1.959E + 01	2.465E + 01	4.915E + 01
F3	Mean	**6.000E + 02**	6.601E + 02	6.127E + 02	6.006E + 02	6.303E + 02	6.032E + 02	6.006E + 02	6.439E + 02
	STD	3.789E-05	9.360E + 00	7.498E + 00	5.542E-01	4.215E + 00	2.224E + 00	1.057E + 00	5.359E + 00
F4	Mean	**8.162E + 02**	9.073E + 02	8.928E + 02	8.380E + 02	9.179E + 02	8.397E + 02	8.322E + 02	9.123E + 02
	STD	**8.047E + 00**	2.812E + 01	2.816E + 01	1.510E + 01	9.070E + 00	8.281E + 00	7.795E + 00	1.010E + 01
F5	Mean	**9.000E + 02**	2.754E + 03	3.020E + 03	**9.000E + 02**	1.505E + 03	1.002E + 03	1.214E + 03	1.639E + 03
	STD	8.571E-02	8.020E + 02	1.048E + 03	2.830E-02	1.968E + 02	1.103E + 02	2.525E + 02	2.933E + 02
F6	Mean	**1.801E + 03**	5.942E + 03	7.265E + 06	9.905E + 03	5.734E + 07	7.701E + 04	1.887E + 03	1.794E + 07
	STD	1.984E + 00	5.884E + 03	1.533E + 07	7.770E + 03	3.785E + 07	1.458E + 05	4.319E + 01	8.196E + 06
F7	Mean	2.021E + 03	2.159E + 03	2.120E + 03	2.065E + 03	2.093E + 03	2.049E + 03	2.037E + 03	2.117E + 03
	STD	3.707E + 00	4.464E + 01	5.724E + 01	3.706E + 01	1.328E + 01	1.980E + 01	1.015E + 01	1.632E + 01
F8	Mean	2.219E + 03	2.280E + 03	2.242E + 03	2.268E + 03	2.243E + 03	2.227E + 03	2.220E + 03	2.241E + 03
	STD	2.174E + 00	5.661E + 01	1.771E + 01	6.575E + 01	3.161E + 00	6.322E + 00	1.980E + 00	3.604E + 00
F9	Mean	2.481E + 03	2.481E + 03	2.486E + 03	2.481E + 03	2.523E + 03	2.495E + 03	2.481E + 03	2.613E + 03
	STD	1.689E-13	3.929E-01	8.621E + 00	6.874E-03	1.505E + 01	1.419E + 01	1.516E-12	2.903E + 01
F10	Mean	**2.500E + 03**	3.865E + 03	3.880E + 03	3.684E + 03	2.512E + 03	3.176E + 03	2.555E + 03	2.534E + 03
	STD	**3.529E-02**	1.047E + 03	1.003E + 03	6.022E + 02	3.829E + 00	6.499E + 02	8.997E + 01	1.369E + 01
F11	Mean	2.993E + 03	2.921E + 03	4.180E + 03	2.930E + 03	3.936E + 03	3.172E + 03	**2.870E + 03**	4.692E + 03
	STD	2.537E + 01	4.148E + 01	8.164E + 02	4.820E + 01	2.914E + 02	2.414E + 02	**1.494E + 02**	5.181E + 02
F12	Mean	2.938E + 03	3.025E + 03	2.952E + 03	2.943E + 03	3.004E + 03	2.961E + 03	3.027E + 03	**2.900E + 03**
	STD	4.240E + 00	7.158E + 01	6.788E + 00	8.017E + 00	1.937E + 01	1.515E + 01	2.920E + 01	**9.714E-05**

Table 10 continued

Function	Stats	EAO	WOA	MFO	MVO	SCA	GWO	WSO	HGSO
	Win	**6**	**0**	**0**	**2**	**0**	**0**	**2**	**1**
Function	Stats	SHO	HLOA	FDA	GTO	GMO	PSO	DE	
F1	Mean	9.621E + 03	3.028E + 02	**3.000E + 02**	**3.000E + 02**	**3.000E + 02**	**3.000E + 02**	**3.000E + 02**	
	STD	4.303E + 03	6.424E + 00	1.504E-13	7.550E-12	7.774E-12	5.171E-14	**0.000E + 00**	
F2	Mean	6.622E + 02	4.534E + 02	4.299E + 02	**4.176E + 02**	4.229E + 02	4.240E + 02	4.477E + 02	
	STD	1.265E + 02	2.666E + 01	2.485E + 01	**2.256E + 01**	2.411E + 01	2.243E + 01	2.008E + 00	
F3	Mean	6.176E + 02	6.588E + 02	6.021E + 02	6.250E + 02	6.210E + 02	6.178E + 02	**6.000E + 02**	
	STD	8.113E + 00	8.198E + 00	1.704E + 00	1.196E + 01	1.541E + 01	8.956E + 00	**8.444E-14**	
F4	Mean	8.878E + 02	9.144E + 02	8.769E + 02	8.712E + 02	8.585E + 02	8.596E + 02	8.206E + 02	
	STD	1.312E + 01	4.170E + 01	1.989E + 01	1.978E + 01	2.826E + 01	1.715E + 01	7.651E + 00	
F5	Mean	2.193E + 03	2.341E + 03	1.211E + 03	1.657E + 03	1.560E + 03	1.107E + 03	**9.000E + 02**	
	STD	1.610E + 02	3.237E + 02	1.793E + 02	5.081E + 02	5.870E + 02	3.083E + 02	**0.000E + 00**	
F6	Mean	1.755E + 06	5.123E + 03	1.845E + 03	1.887E + 03	1.928E + 03	5.852E + 03	**1.801E + 03**	
	STD	5.504E + 06	5.143E + 03	1.846E + 01	4.309E + 01	1.172E + 02	4.257E + 03	**5.442E-01**	
F7	Mean	2.096E + 03	2.244E + 03	2.058E + 03	2.105E + 03	2.101E + 03	2.071E + 03	**2.004E + 03**	
	STD	3.661E + 01	1.168E + 02	1.948E + 01	2.186E + 01	3.819E + 01	3.651E + 01	**8.192E + 00**	
F8	Mean	2.225E + 03	2.536E + 03	2.224E + 03	2.229E + 03	2.240E + 03	2.260E + 03	**2.207E + 03**	
	STD	1.697E + 00	2.186E + 02	2.832E + 00	7.828E + 00	3.718E + 01	6.238E + 01	**9.409E + 00**	
F9	Mean	2.530E + 03	2.481E + 03	2.481E + 03	2.481E + 03	2.481E + 03	**2.465E + 03**	2.481E + 03	
	STD	2.285E + 01	8.048E + 00	3.713E-13	4.171E-04	4.575E-03	**1.068E-12**	0.000E + 00	
F10	Mean	2.961E + 03	4.987E + 03	2.659E + 03	2.547E + 03	2.580E + 03	3.663E + 03	2.507E + 03	
	STD	5.142E + 02	1.148E + 03	3.707E + 02	7.558E + 01	1.592E + 02	7.581E + 02	2.702E + 01	
F11	Mean	5.010E + 03	2.943E + 03	2.910E + 03	2.930E + 03	2.930E + 03	2.897E + 03	2.947E + 03	
	STD	6.567E + 02	1.772E + 02	3.162E + 01	4.830E + 01	4.830E + 01	1.098E + 02	5.074E + 01	

Table 10 continued

Function	Stats	SHO	HLOA	FDA	GTO	GMO	PSO	DE
F12	Mean	3.036E + 03	3.187E + 03	2.955E + 03	2.968E + 03	2.966E + 03	3.036E + 03	2.938E + 03
	STD	4.212E + 01	1.163E + 02	1.123E + 01	3.131E + 01	3.268E + 01	1.621E + 02	4.040E + 00
	Win	**0**	**0**	**1**	**2**	**1**	**2**	**6**

Bold values represent the best results

Table 9, for $d = 10$, EAO achieves 8 wins out of 12 benchmark functions, outperforming all other optimizers. The CEC2022 dataset presents a range of challenging function characteristics, such as high multimodality, non-separability, and composite structures, that rigorously test algorithmic capabilities.

EAO's robust performance in this setting shows its ability to effectively navigate complex search landscapes, consistently avoiding local optima and efficiently exploring the search space. Notably, DE follows with 6 wins, while other competitive optimizers like GMO, GTO, and PSO secure fewer wins, further highlighting EAO's advantage.

In higher-dimensional problems ($d = 20$), as shown in Table 10, EAO maintains good performance, securing 6 wins out of 12 benchmark functions. In particular, EAO and DE achieve the highest number of wins, demonstrating their effectiveness in handling the increased complexity introduced by higher dimensionality. Challenges such as enhanced non-separability and more complex landscapes often limit an algorithm's scalability and efficiency, yet EAO continues to excel under these conditions. This performance highlights its robustness and suitability for large-scale optimization tasks, where maintaining a balance between exploration and exploitation is essential.

The results of the Wilcoxon signed-rank test to compare EAO with other optimizers on the IEEE CEC2022 benchmark are presented in Table 11. For the UN (unimodal and nonseparable) functions, EAO demonstrates superior performance, significantly outperforming WOA, MFO, MVO, SCA, GWO, HGSO, SHO, HLOA, and GTO. In the MM (multimodal) category, EAO outperforms all other optimizers, including WOA, MFO, MVO, SCA, GWO, WSO, HGSO, SHO, FDA, and GTO; however, GMO is statistically better than EAO, as indicated by a 0/1/2 result.

For the HF (hybrid) functions, EAO achieves strong performance, statistically outperforming most of the compared algorithms. In CF (composition) functions, EAO again excels by significantly outperforming WOA, MFO, MVO, SCA, GWO, HGSO, SHO, and GTO. Moreover, EAO outperforms PSO with a 12/0/0 record, while maintaining a strong 7/3/2 outcome against DE. For a complete view of the p-values for each function and optimizer, refer to Table 21 in Appendix A.

In addition, the execution time results of EAO and the other compared optimizers on the CEC2022 benchmark functions with ($d = 10$), as shown in Table 12, demonstrate that EAO maintains good efficiency compared to other optimization algorithms. EAO achieves its results within a reasonable computational time, making it a practical choice to solve complex optimization problems without excessive execution overhead.

6 Visual analysis of EAO

This section provides a comprehensive visual analysis of the EAO using a variety of charts, including sensitivity analysis, search history, trajectory, and average fitness evaluation. Additionally, the convergence behavior of EAO is analyzed and

Table 11 Statistical results of Wilcoxon signed-rank test for EAO versus other compared optimizers over the IEEE CEC2022 benchmark dataset, Dim=10, Runs=30, and Iterations=10,000

Function	EAO versus WOA	EAO versus MFO	EAO versus MVO	EAO versus SCA
UN	2/0/0	1/0/1	2/0/0	2/0/0
MM	3/0/0	3/0/0	2/0/1	3/0/0
HF	3/0/0	3/0/0	2/0/1	3/0/0
CF	4/0/0	3/0/1	4/0/0	4/0/0
Total	12/0/0	10/0/2	10/0/2	12/0/0
Function	**EAO versus GWO**	**EAO versus WSO**	**EAO versus HGSO**	**EAO versus SHO**
UN	2/0/0	1/1/0	2/0/0	2/0/0
MM	2/0/1	2/1/0	3/0/0	3/0/0
HF	3/0/0	1/2/0	3/0/0	3/0/0
CF	4/0/0	3/0/1	4/0/0	4/0/0
Total	11/0/1	7/4/1	12/0/0	12/0/0
Function	**EAO versus GMO**	**EAO versus HLOA**	**EAO versus FDA**	**EAO versus GTO**
UN	0/2/0	1/0/1	0/1/1	1/1/0
MM	0/1/2	3/0/0	2/1/0	3/0/0
HF	3/0/0	3/0/0	2/0/1	3/0/0
CF	3/0/1	3/0/1	2/1/1	2/1/1
Total	6/3/3	10/0/2	6/3/3	9/2/1
Function	**EAO versus DE**	**EAO versus PSO**		
UN	2/0/0	2/0/0		
MM	2/1/0	3/0/0		
HF	1/2/0	3/0/0		
CF	2/0/2	4/0/0		
Total	7/3/2	12/0/0		

compared against other compatative optimizers to highlight its performance advantages. The analysis concludes with an exploration and exploitation assessment, providing insights into its adaptability across diverse optimization scenarios.

6.1 Sensitivity analysis of EAO parameters

As can be seen in Fig. 6, the sensitivity analysis of the EAO optimizer reveals that it exhibits robust performance for most functions, consistently reaching their respective optimal values with minimal variability in combinations of EC parameters and substrates. Functions such as F1, F3, F6, F7, F8, F9, and F12 demonstrate high stability, achieving their optimal values with EC less than 0.3.

Table 12 Comparison of execution times for optimizers on the CEC 2022 benchmark dataset (Dim=10, Iterations=1000)

Fun.	EAO	WOA	MFO	MVO	SCA	GWO	WSO	HGSO	SHO	HLOA	FDA	GTO	GMO	DE	PSO
F1	0.45	0.24	0.29	0.62	0.26	0.30	0.24	1.92	0.69	1.06	0.55	0.63	4.21	0.37	0.26
F2	0.46	0.17	0.21	0.28	0.21	0.20	0.15	1.74	0.47	0.83	0.46	0.48	4.36	0.30	0.18
F3	0.61	0.27	0.31	0.33	0.26	0.29	0.23	2.12	0.60	0.92	0.63	0.66	3.91	0.40	0.29
F4	0.44	0.19	0.25	0.29	0.19	0.22	0.17	1.75	0.51	0.87	0.51	0.52	4.57	0.32	0.21
F5	0.47	0.20	0.25	0.31	0.25	0.24	0.18	1.76	0.52	0.86	0.53	0.54	4.35	0.33	0.22
F6	0.40	0.17	0.22	0.27	0.22	0.24	0.15	1.73	0.47	0.84	0.46	0.48	4.22	0.30	0.18
F7	0.69	0.31	0.36	0.43	0.36	0.39	0.31	1.91	0.65	0.96	0.70	0.71	4.21	0.48	0.37
F8	0.79	0.36	0.41	0.39	0.33	0.34	0.29	1.89	0.73	1.14	0.83	0.98	3.99	0.55	0.45
F9	0.66	0.29	0.30	0.33	0.28	0.30	0.20	1.82	0.63	0.99	0.84	0.86	4.05	0.43	0.32
F10	0.62	0.27	0.32	0.38	0.33	0.35	0.26	1.83	0.61	0.92	0.63	0.65	4.17	0.40	0.30
F11	0.82	0.38	0.43	0.42	0.34	0.36	0.31	1.93	0.91	1.16	1.00	0.84	4.16	0.51	0.40
F12	0.84	0.39	0.36	0.41	0.36	0.38	0.28	1.89	0.74	1.02	0.84	0.85	4.68	0.56	0.46

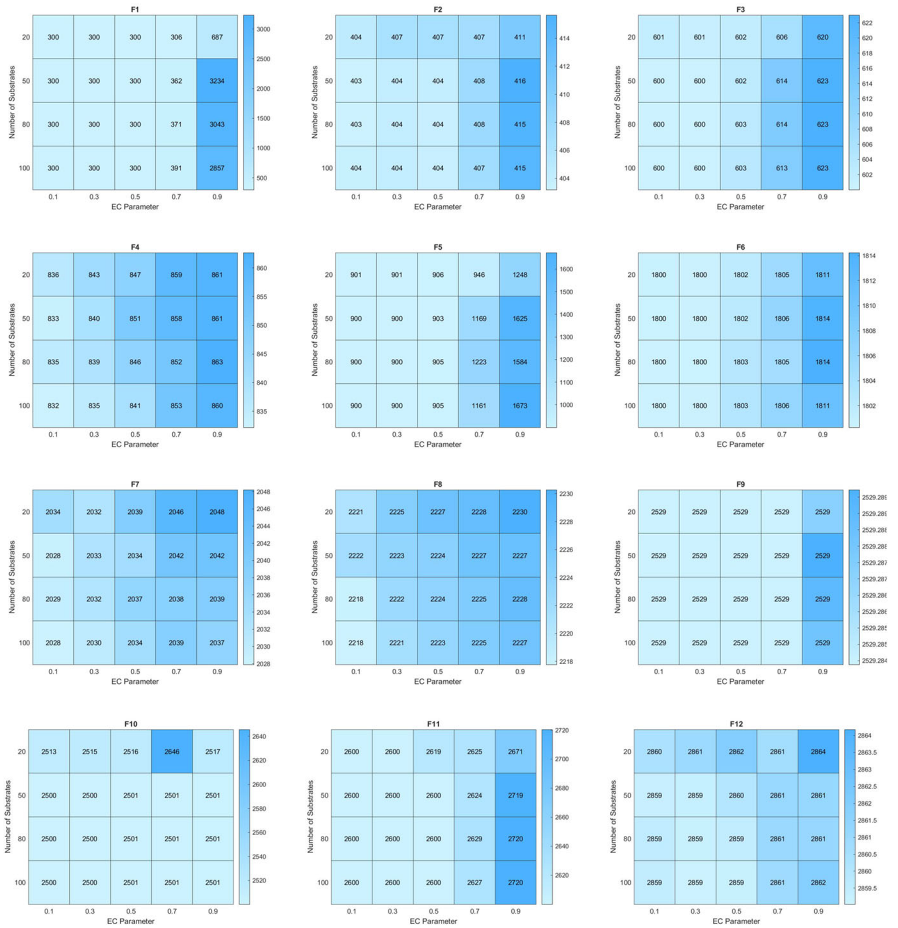

Fig. 6 Sensitivity analysis of EAO parameters (F1–F12)

In contrast, functions like F2, F4, F5, F10, and F11 display slight to moderate sensitivity, where optimal performance depends on specific EC parameter and substrate settings. However, even for these functions, the optimizer reliably approaches the optimal values under appropriate conditions. EAO optimizer proves to be effective and stable, with only minor tuning required for a few functions to achieve optimal performance.

6.2 EAO search history, trajectory, and average fitness analysis

This section analyzes the search history, trajectory of the first substrate, and average fitness of the EAO optimizer. The second column in Figs. 7 and 8 illustrate the search history of EAO substrates throughout the iterations. As can be seen in the figure, EAO exhibits a consistent pattern across the tested functions, where substrates continuously explore promising areas of the search space and subsequently exploit regions in close proximity to the global optima with high precision. These observations ensure the effectiveness of EAO in reaching the global optimum.

For the selected classical 23 functions (F1, F4, F6, F10, F11, F13, and F18), in Fig. 7 the search history of the EAO substrates, highlighting their ability to navigate the search space by oscillating between exploration and exploitation phases. This adaptive behavior ensures continuous improvement in fitness, as the substrates refine their positions to move closer to the global optima. The trajectory plots of the first substrate reveal a rapid reduction in movement magnitude over time, suggesting that after an initial broad search, EAO effectively focus its efforts in more promising areas of the search domain. Moreover, the average fitness curves illustrate the optimizer's capacity to systematically reduce the overall fitness values.

On the other hand, when evaluating the CEC2022 benchmark dataset (F1-F3, F11, and F12), it can be seen in Fig. 8 which contain a more challenge problems with various complexity, where EAO maintained similarly good results. The second column shows the history around the global optimum which confirms EAO's capacity for robust exploration. Moreover, in the third column, the substrate trajectories confirm the success of EAO shifting from exploration to a more localized exploitation phase. Concurrently, the average fitness plots also show improvements, signifying that EAO effectively extracts the best possible solutions in these more challenging CEC2022 scenarios. These observations collectively highlight EAO's adaptability and effectiveness, whether tested against the classical 23 functions or the more demanding CEC2022 benchmarks.

Moreover, Figs. 9 and 10 illustrate the 3D trajectories of the first substrate of EAO over the CEC2022 benchmark (F1-F12). EAO demonstrates an adaptive balance between exploration and exploitation. Initially, the substrate moves in large steps, exploring large areas of the search space. As the iterations progress, the trajectories become more concentrated, with movements focusing on specific regions, indicating a transition to targeted exploitation. This movement shows that EAO effectively adjusts the substrate's position, starting with broad exploration to identify promising regions, followed by refined searches around the global optima to avoid premature convergence.

6.3 Convergence analysis of EAO with competitive optimizers

EAO demonstrates consistently superior performance across the selected benchmark functions when compared to the other optimizers, as shown by its rapid convergence and lower final fitness values in most of the test cases (see Fig. 11). From the full-range plots, EAO exhibits a sharper descent curve in the early iterations, indicating a strong global exploration capability, while the zoomed-in views around the optima further highlight its robust exploitation behavior by refining solutions more effectively than alternative methods.

In particular, on most of the tested functions F1-F4 and F9-F11, EAO achieves the best observed fitness levels within fewer iterations, showing its efficiency to tackle both unimodal and multimodal problems. Even in more challenging

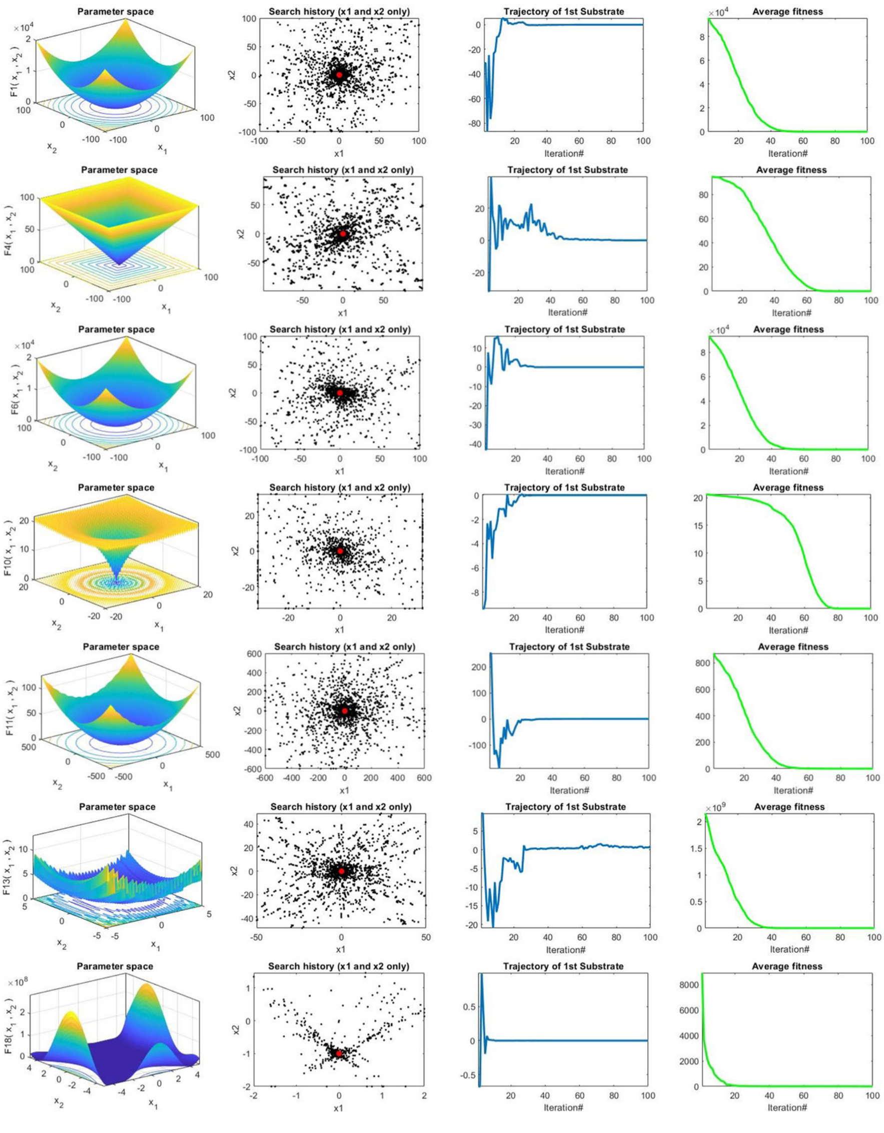

Fig. 7 Illustration of the search history, trajectory of the first substrate, and average fitness of EAO optimizer over selected functions F1, F4, F6, F10, F11, F13, and F18 of the 23 classical benchmark functions

landscapes like F14, EAO consistently maintains a convergence advantage, demonstrating superior stability and adaptability. These results indicate that EAO is a competitive and reliable choice among bio-inspired and nature-inspired optimization techniques that effectively tackle a wide range of complex search spaces.

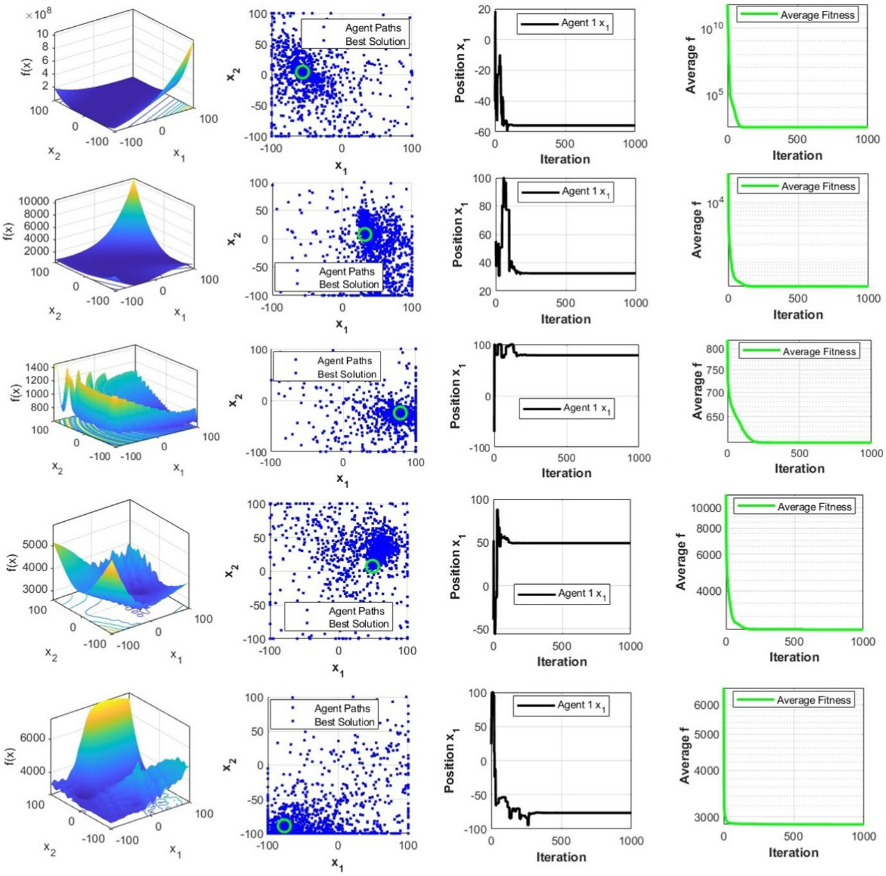

Fig. 8 Illustration of the search history, trajectory of the first substrate, and average fitness of EAO optimizer over selected functions F1-F3, F11, and F12 (CEC2022)

Moreover, across the selected CEC2022 benchmark functions (F1–F8) as shown in Fig. 12, EAO demonstrates a consistently competitive and often superior convergence trend in comparison with the other competitive optimizers. Specifically, it combines robust global exploration with a effective local exploitation phase, often achieving lower fitness values in fewer iterations. In functions such as F1, F5, and F6, EAO curve decreases rapidly from the beginning, demonstrating its capacity to efficiently identify promising regions and refine candidate solutions toward the global minimum. Even in more challenging landscapes like F7 and F8, EAO maintains a strong convergence pattern, either outperforming or performing comparably to the other competitive optimizers such as MVO, WSO, FDA, GTO, HLOA, and SHO, indicating a high level of adaptability.

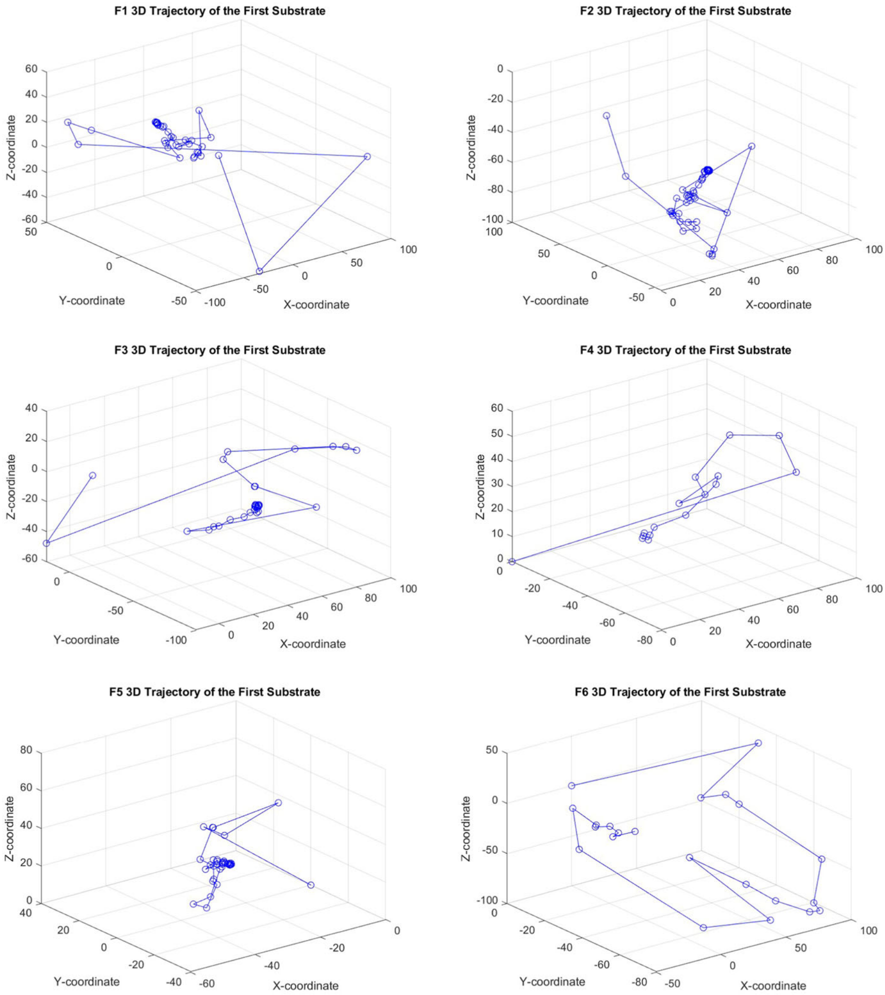

Fig. 9 EAO 3D trajectory analysis over selected function of CEC2022 (F1-F6)

6.4 EAO exploration and exploitation analysis

This section provides a detailed examination of how the EAO algorithm manages the trade-off between exploration and exploitation. We highlight how EAO adapts its strategy over time to scan broad regions initially and subsequently refine solutions within more promising areas. This balance reduces the risk of premature convergence and ensures robust performance across diverse optimization landscapes.

As it can be seen in Figs. 13, 14, and 15 which illustrate the dynamic balance between exploration and exploitation across various benchmark functions. The vertical axis in these figures represents the percentage of EAO effort dedicated to

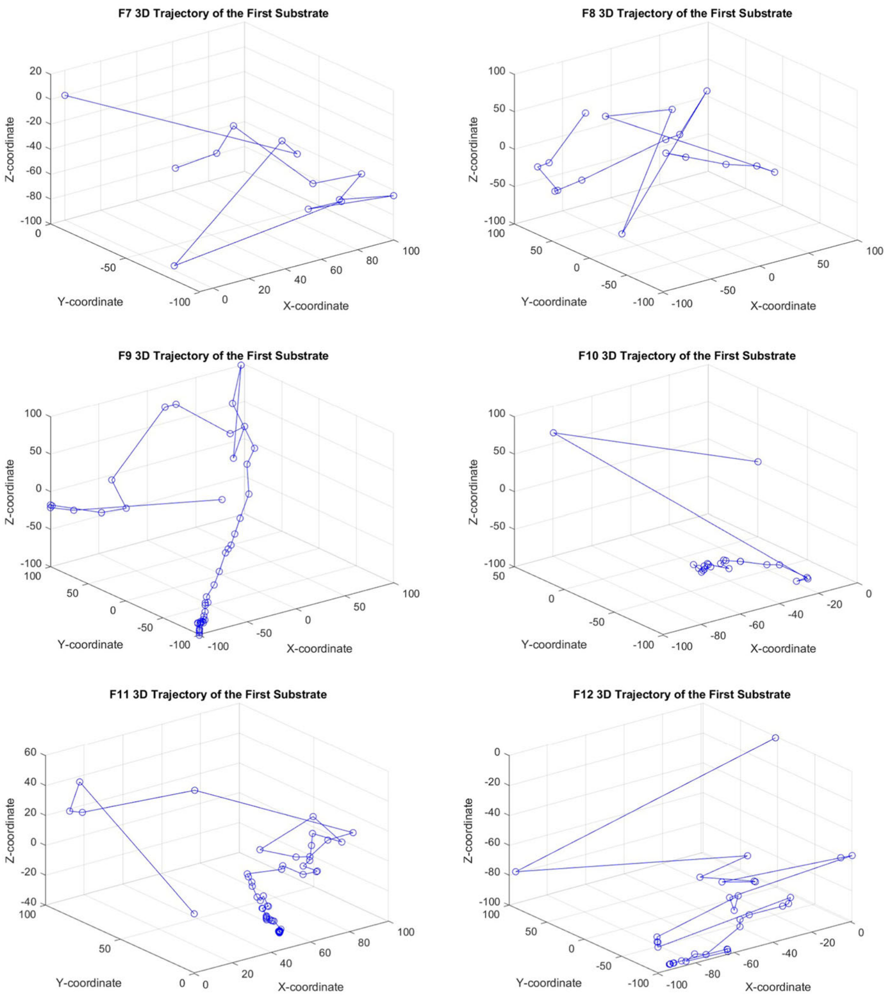

Fig. 10 EAO 3D trajectory analysis over selected function of CEC2022 (F7-F12)

either exploration or exploitation at each iteration, while the horizontal axis shows the iteration count.

Initially, the exploration percentage is high, indicating that EAO allocates a substantial part of its computational steps toward searching a broad and diverse region of the solution space. This early emphasis on exploration ensures that EAO samples multiple areas widely, reducing the likelihood of premature convergence to suboptimal regions. The corresponding high exploration curve visually confirms that at the beginning, a significant fraction of EAO's operations are geared toward trying new substrates, applying broader movements, and maintaining high diversity among candidate solutions.

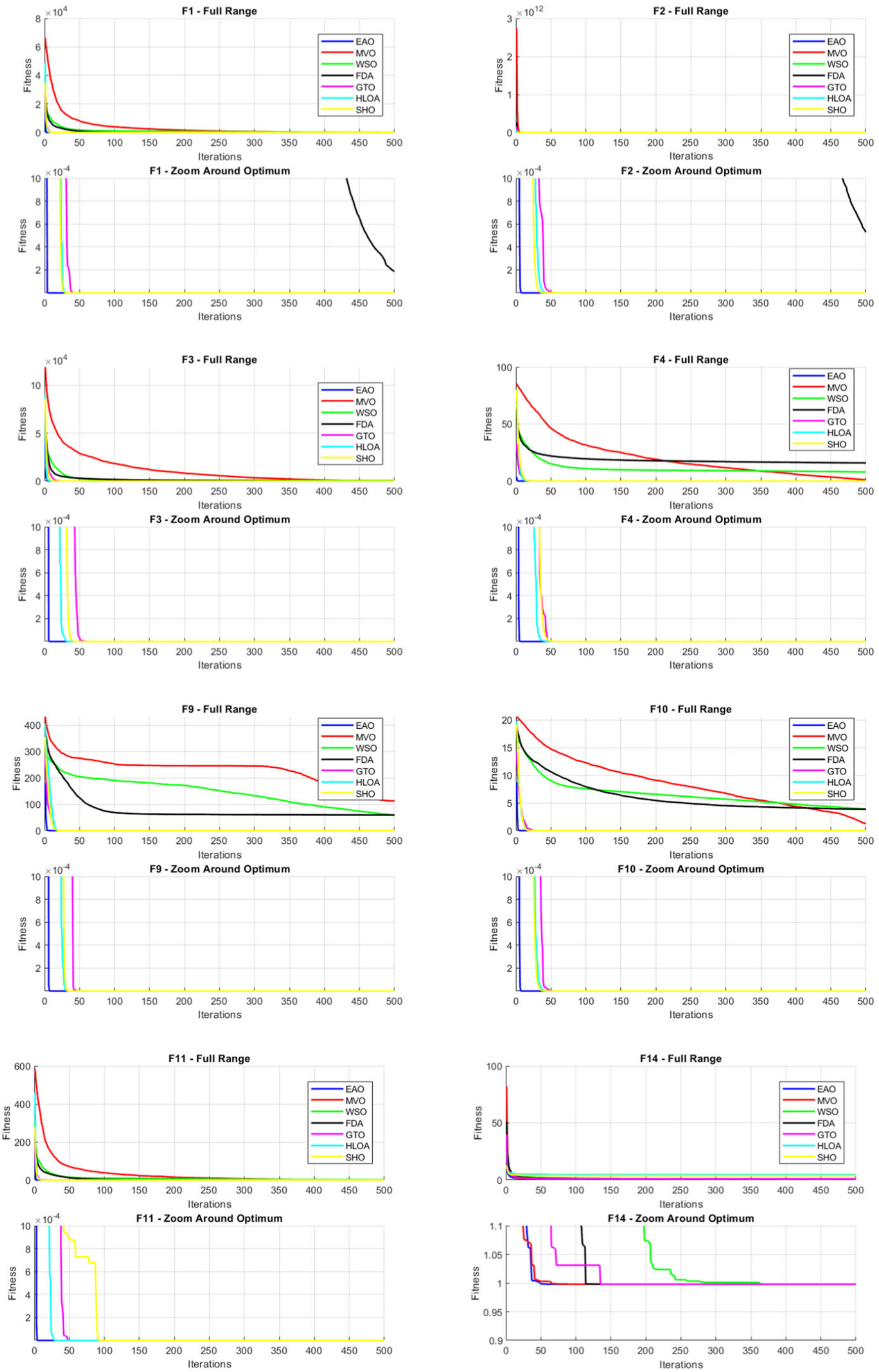

Fig. 11 Convergence curve analysis over selected functions from the classical 23 benchmark functions (F1-F4, F9-F11, and F14)

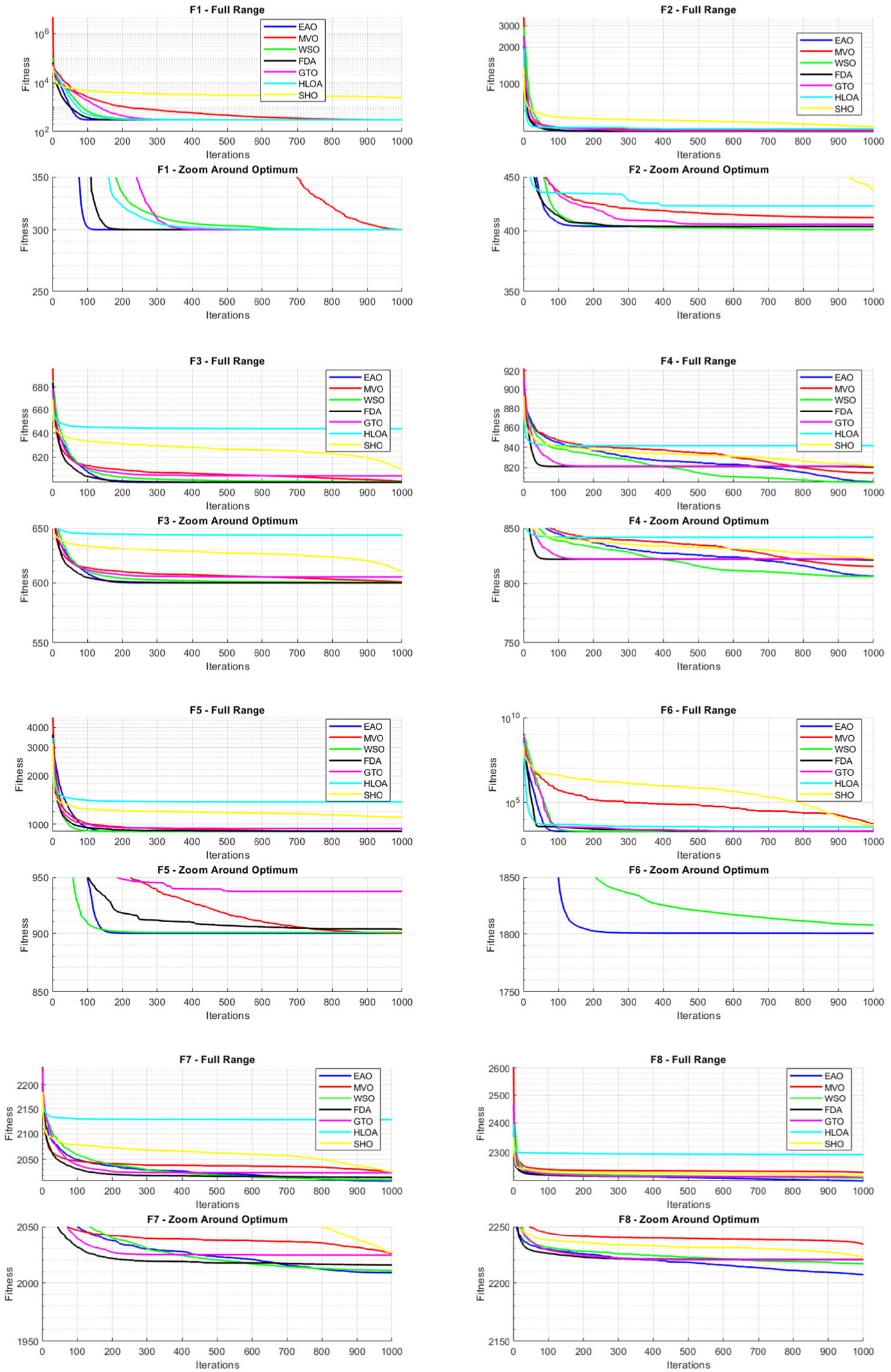

Fig. 12 Convergence curve analysis over CEC2022 benchmark functions (F1–F8)

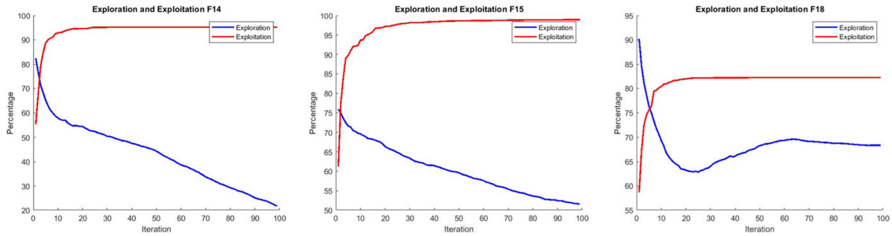

Fig. 13 Illustration of exploration and exploitation over selected functions (F14, F15, F18) from the 23 benchmark functions

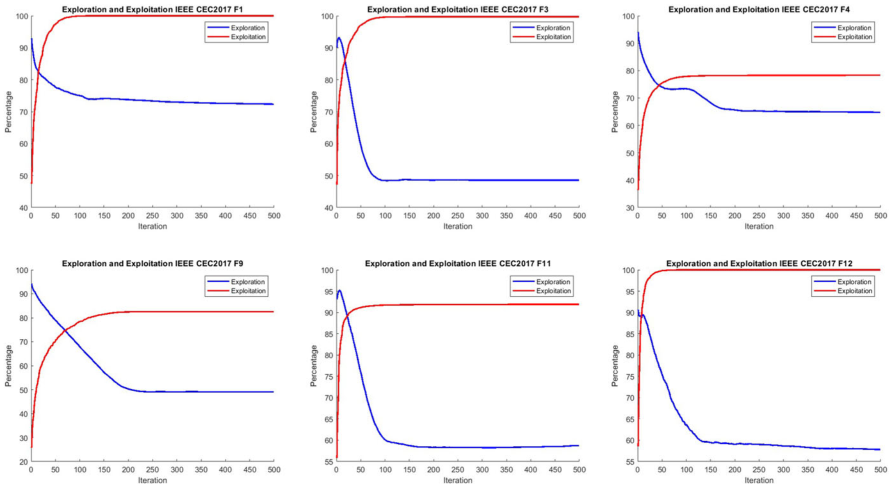

Fig. 14 Illustration of exploration and exploitation over selected functions (F1, F3, F4, F9, F11, F12) from the CEC2017 benchmark functions

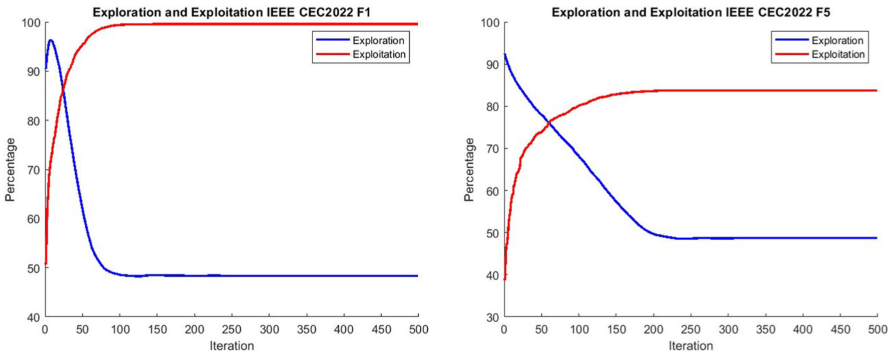

Fig. 15 Illustration of exploration and exploitation over selected functions (F1, F5) from the CEC2022 benchmark functions

As iterations progress, the exploration percentage typically decreases. This decrease indicates that EAO is progressively refining its search strategy. Instead of continually exploring distant areas, EAO begins to focus more on regions where it has identified higher-quality solutions. Consequently, the exploitation percentage rises, reflecting an increased investment of computational resources in increasing the search around promising substrates. Exploitation-oriented steps may involve localized refinements, employing strategies that employing previously found high-quality solutions. In other words, EAO shifts from a wide-ranging search (high exploration) to a more targeted, solution-enhancing approach (high exploitation).

The calculation of these percentages involves assessing the relative proportion of exploration versus exploitation actions at each iteration. For instance, exploration actions might include large sinusoidal updates, random perturbations, or the introduction of new substrates from unexplored regions, while exploitation actions might involve small, controlled steps, selective improvements guided by the best solutions found so far, or parameters that narrow the search around a localized promising area. By quantifying the number and intensity of exploration-based updates against exploitation-based refinements, EAO can determine the current percentages.

The rise of the exploitation curve, coupled with the fall of the exploration curve, indicates that EAO is sucess in the initial search phase. Over time, as the exploitation percentage stabilizes at a high level, it suggests that EAO has targeted to a smaller region of the search space and is actively refining solutions there. This stable, high-exploitation state generally corresponds to the later stages of the optimization process, where EAO intensively move to the best-found solutions to achieve minimal error or maximal fitness, depending on the objective.

6.4.1 Ablation study on EAO exploration and exploitation components

In this ablation study, we assess the impact of EAO's exploration and exploitation mechanisms by comparing performance against variants in which these components are partially or entirely disabled. The standard EAO approach balances exploration and exploitation by prioritizing early exploration to cover extensive regions of the search space, followed by a shift toward exploitation to refine promising solutions. Altered variants, including 'No Exploration,' 'No Exploitation,' and 'No Exploration and No Exploitation (static balance),' highlight the importance of each component, where exploration drives global search capability, while exploitation confirms final precision.

Figures 16 and 17 show the comparative results on CEC 2017 benchmarks ($F1$–$F5$) for dimensions ($d = 10$ and $d = 100$) indicate that the original EAO performs well in both ($d = 10$ and $d = 100$). In contrast, other variants show significant shortcomings on $F1$–$F5$. Specifically, 'EAO with Exploration only' struggles on these functions with lower performance and stability. Similarly, 'EAO with Exploitation only' fails to match the original EAO on several benchmarks and suffers from higher variance, particularly on $F2$ and $F3$. The 'No Exploration and No Exploitation' variant consistently performs poorly, showing substantial variability across all functions. These observations show the necessity of robust exploration and exploitation mechanisms, as well as their dynamic coordination, in delivering high-quality optimization results.

7 EAO for engineering design problems

This section discuss solving of different engineering design problems using EAO and compares the results with those of well-known and state-of-the-art optimizers. The effectiveness of the EAO optimizer is evaluated through its application to a series of engineering problems (see Table 13). These problems consist of different constraints and objectives and range from reducing the weight of a Three-bar truss to minimizing the cost of a Pressure Vessel.

The engineering problems assess the EAO optimizer's ability to navigate complex optimization landscapes and obtain optimal solutions. In all experiments in this section, we used 500 iterations, 30 substrate/agents and 30 runs.

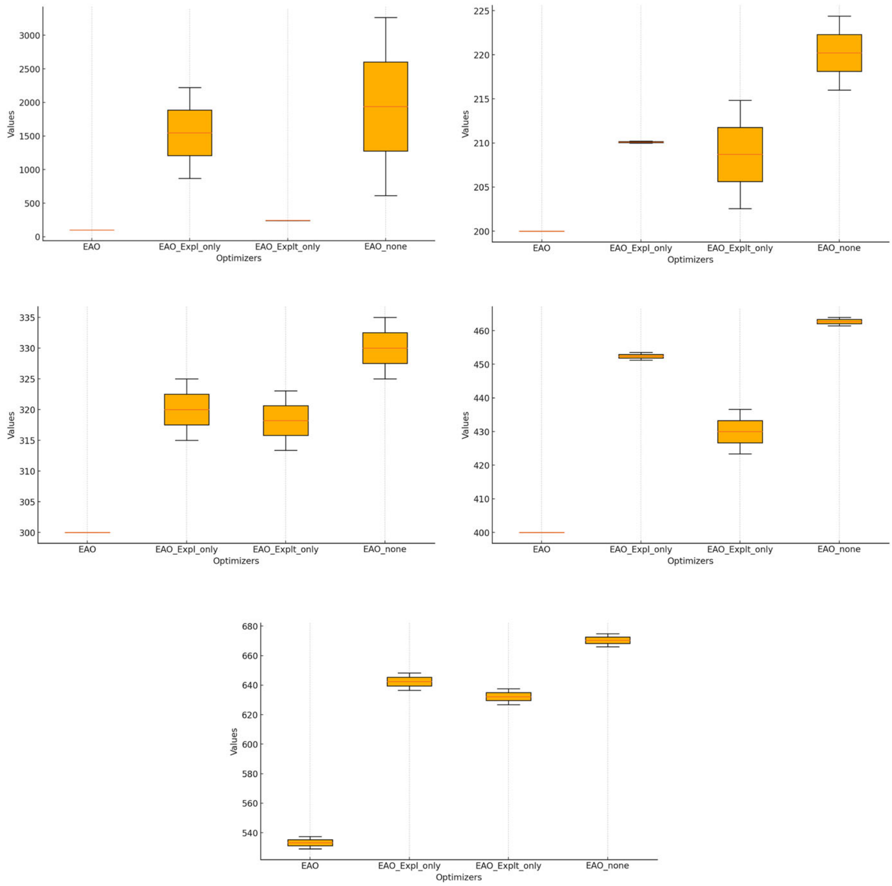

Fig. 16 Ablation study on EAO exploration and exploitation components (F1–F5) at dim = 10

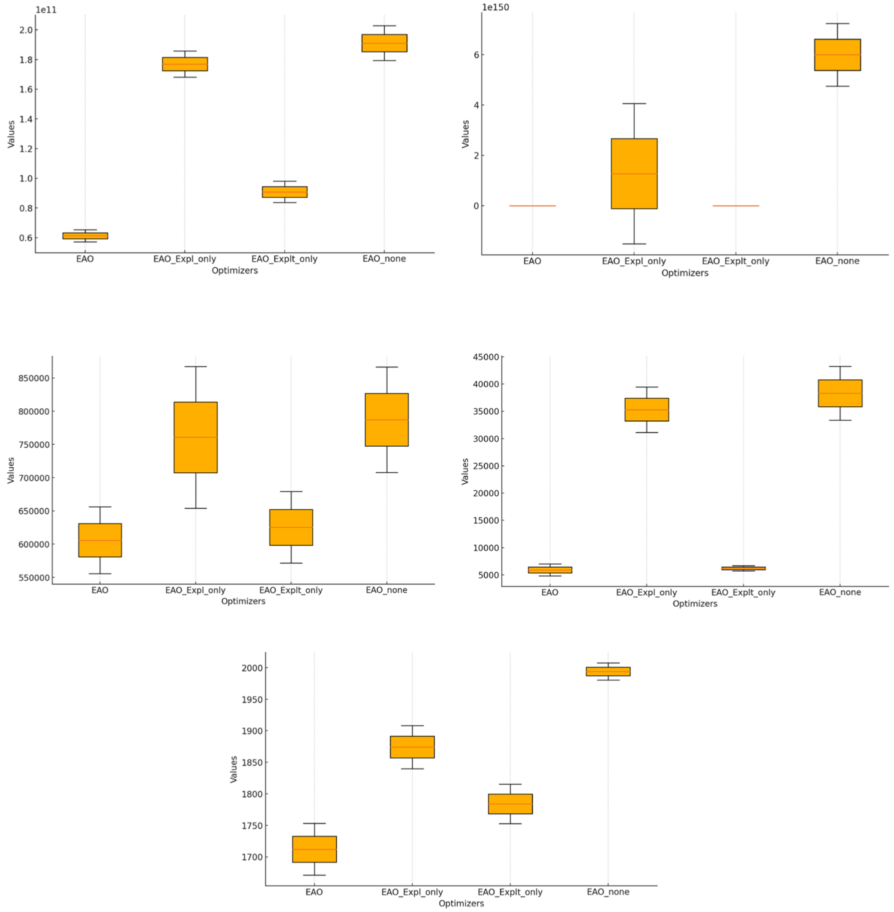

Fig. 17 Ablation study on EAO exploration and exploitation components (F1–F5) at dim = 100

NV, number of variables; NCV, number of continuous variables; NDV, number of discrete variables; NC, number of constraints; NIC, number of inequality constraints; NAC, number of active constraints; F/S, ratio between the feasible solutions in the search space (F) and the entire search space (S); DO, design objective.

7.1 Pressure vessel design

The pressure vessel design problem (see Fig. 18) is a well-known optimization problem in mechanical engineering. The goal is to minimize the manufacturing cost of a cylindrical pressure vessel with hemispherical ends. The design variables include the thickness of the shell, the thickness of the head, the inner radius, and the length of the cylindrical section, which influence the cost, safety, and functionality of the pressure vessel.

Table 13 Characteristics of engineering problems

No.	Case name	NV	NCV	NDV	NC	NIC	NAC	F/S	DO
1	Three-bar truss	2	0	0	3	3	–	–	Minimum weight
2	Cantilever beam	5	5	0	1	1	–	–	Minimum weight
3	Tension/compression spring	3	3	0	4	4	2	0.1	Minimum weight
4	Pressure vessel	4	2	2	4	4	2	0.40	Minimum cost
5	Speed reducer	7	6	1	11	11	3	0.004	Minimum weight

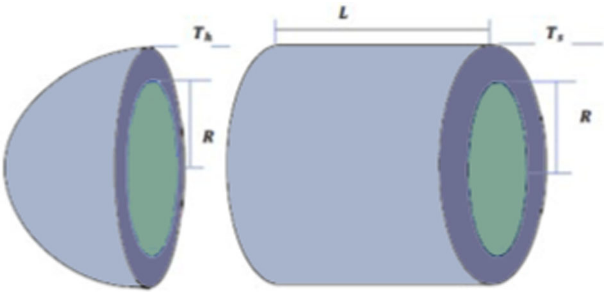

Fig. 18 Pressure vessel design

Mathematical formulation

The cost function consists of material costs and forming costs and is expressed as the sum of these two components, as shown in Eq. 18 - 20:

$$f_{\text{material}}(\mathbf{x}) = 0.6224 \times 0.0625 \times x_1 \times x_3 \times x_4 + 1.7781 \times 0.0625 \times x_2 \times x_3^2 \quad (18)$$

$$f_{\text{forming}}(\mathbf{x}) = 3.1661 \times (0.0625 \times x_1)^2 \times x_4 + 19.84 \times (0.0625 \times x_1)^2 \times x_3 \quad (19)$$

$$f(\mathbf{x}) = f_{\text{material}}(\mathbf{x}) + f_{\text{forming}}(\mathbf{x}) \quad (20)$$

where x_1 is the shell thickness, x_2 is the head thickness, x_3 is the inner radius, and x_4 is the length of the cylindrical section. The aim is to minimize the total cost of the vessel while satisfying a set of physical and practical constraints to ensure structural integrity and compliance as shown in Eq. 21 – 24:

$$-0.0625 \times x_1 + 0.0193 \times x_3 \leq 0 \quad (21)$$

$$-0.0625 \times x_2 + 0.00954 \times x_3 \leq 0 \quad (22)$$

Table 14 Optimization results of the pressure vessel design with optimal variable values (X1-X4)

Optimizer	Mean	Std	Max	Best	X1	X2	X3	X4	Rank
EAO	5885.341	0.002681	5885.346	5885.333	12.45070	6.15439	40.3196	200	2
WOA	8443.758	1648.911	13426.47	6467.621	14.77863	8.38955	47.7725	117.0631	15
MFO	6446.669	487.8125	7319.001	5885.333	12.45070	6.15439	40.3196	200	10
MVO	6624.325	350.8114	7301.786	5987.446	13.18830	6.59187	42.6855	169.5352	11
SCA	7023.131	638.4989	8476.98	6096.201	12.68371	6.65397	40.8625	196.4697	12
GWO	5962.382	194.6747	6862.838	5891.576	12.47024	6.17605	40.3706	199.3069	4
WSO	5885.421	0.625682	5889.757	5885.333	12.45070	6.15439	40.3196	200	3
HGSO	7502.732	371.5523	8385.03	6625.429	16.35427	8.41188	52.8355	79.1045	13
SHO	7707.929	429.6275	8475.842	6225.151	13.44748	6.91504	43.1143	169.1187	14
HLOA	6374.745	407.4204	7247.96	5902.704	12.61122	6.23374	40.8395	192.8879	8
FDA	6330.634	326.2833	7263.877	5885.485	12.45212	6.15510	40.3242	199.9357	7
GTO	6183.485	467.6585	7319.001	5885.333	12.45070	6.15439	40.3196	200	5
GMO	6446.434	453.5686	7318.991	5885.797	12.45504	6.15653	40.3337	199.8043	9
PSO	6285.605	259.6705	7026.126	5960.843	13.12080	6.48562	42.4896	171.8488	6
DE	5885.333	4.61E-08	5885.333	5885.333	12.45070	6.15439	40.3196	200	1

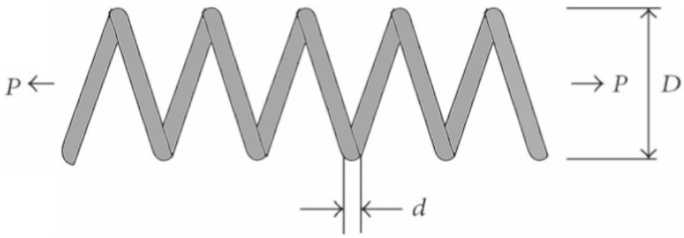

Fig. 19 Tension/compression spring design

$$-\pi \times x_3^2 \times x_4 - \frac{4}{3}\pi \times x_3^3 + 1296000 \le 0 \tag{23}$$

$$x_4 - 240 \le 0 \tag{24}$$

Additionally, the design variables must satisfy minimum size requirements to be manufacturable and to meet safety standards, where x_1 must be at least 1, x_2 must be at least 1, x_3 must be at least 10, and x_4 must be at least 10.

The goal is to minimize the total cost function $f(\mathbf{x})$ subject to the constraints. The formulation includes a penalty for violating any constraint to ensure that only feasible designs are considered during the optimization process.

The results in Table 14 show that DE achieves the best performance, reaching the lowest mean value of 5885.333 with an exceptionally low standard deviation of 4.61×10^{-8} to solve the pressure vessel design problem. However, EAO closely follows DE, ranking second with a mean value of 5885.341 and a very small standard deviation of 0.002681, demonstrating near-identical performance. Both DE and EAO reached the best solution of 5885.333, confirming their ability to converge precisely to the optimal value. Compared to the other optimizers, WSO ranks third, showing slightly higher mean value and standard deviation, while GWO and GTO follow with larger deviations in their results. Alternatively, optimizers such as WOA, SHO, and HGSO exhibit significantly higher mean values and variations, indicating lower consistency and reliability.

7.2 Tension/compression spring design

The spring design problem (see Fig. 19) is a classic optimization problem aimed at minimizing the weight of a tension/compression spring. The design must meet a set of performance constraints, including limits on shear stress, surge frequency, and minimum deflection.

Mathematical formulation The objective of the spring design problem is to minimize the weight of the spring, as shown in Eq. 25:

$$f(\mathbf{x}) = (x_3 + 2)x_2x_1^2 \tag{25}$$

where x_1 is the wire diameter, x_2 is the mean coil diameter, and x_3 is the number of active coils. These variables determine the physical characteristics and performance of the spring, which are controlled by constraints that are derived from the physical properties of the spring, as shown in Eqs. 26–29. These constraints ensure that the spring can withstand the applied forces without failure due to shear stress or excessive deflection and that it has an acceptable resonance frequency.

$$\frac{-x_2^3 x_3}{71785 x_1^4} + 1 \leq 0 \tag{26}$$

$$\frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3 - x_1^4)} + \frac{1}{5108 x_1^2} - 1 \leq 0 \tag{27}$$

$$1 - \frac{140.45 x_1}{x_2^2 x_3} \leq 0 \tag{28}$$

$$\frac{x_1 + x_2}{1.5} - 1 \leq 0 \tag{29}$$

The results in Table 15 highlight the superior performance of EAO in solving the tension/compression spring design problem, achieving the lowest mean value of 0.012668 and the smallest standard deviation of 0.000002. Furthermore, EAO, WSO, and DE all reached the best solution of 0.012665, where WSO and DE follow closely with slightly higher standard deviations. Compared to the other algorithms, GWO and GTO exhibit greater variability in their results, while optimizers such as FDA, MFO, and SCA present higher mean values, indicating reduced consistency and reliability. HLOA and MVO demonstrate the highest variations, further emphasizing their lack of robustness in this problem.

Table 15 Statistical results of the optimization algorithms for the tension/compression spring design with optimal variable values (X1-X3)

Optimizer	Mean	Std	Max	Best	X1	X2	X3	Rank
EAO	0.012668	0.000002	0.012677	0.012665	0.051698	0.356936	11.276513	1
WOA	0.013497	0.000872	0.016945	0.012665	0.051000	0.340155	12.344571	12
MFO	0.013019	0.000484	0.014645	0.012670	0.051000	0.339664	12.403848	8
MVO	0.016411	0.002271	0.018457	0.012691	0.051628	0.355259	11.374979	14
SCA	0.013020	0.000229	0.013725	0.012713	0.051721	0.357479	11.244460	9
GWO	0.012687	0.000026	0.012845	0.012670	0.051825	0.359995	11.099405	4
WSO	0.012668	0.000006	0.012696	0.012665	0.051690	0.356734	11.287991	2
HGSO	0.014305	0.000824	0.016559	0.012834	0.051684	0.356596	11.296086	13
SHO	0.013045	0.000533	0.015855	0.012725	0.051722	0.357511	11.242599	10
HLOA	0.018610	0.004199	0.028939	0.012665	0.051689	0.356718	11.288968	15
FDA	0.012988	0.000838	0.017533	0.012665	0.051000	0.339962	12.377552	7
GTO	0.012710	0.000108	0.013330	0.012665	0.051924	0.362345	10.969224	5
GMO	0.013089	0.000524	0.014721	0.012665	0.052185	0.368761	10.616320	11
PSO	0.012969	0.000452	0.014123	0.012666	0.051881	0.361349	11.022549	6
DE	0.012669	0.000011	0.012724	0.012665	0.051689	0.356718	11.288963	3

7.3 Speed reducer design

The speed reducer design problem (See Fig. 20)is a multifaceted optimization problem that seeks to minimize the weight of a gear train within a speed reducer assembly. The design variables influence the geometry and dimensions of the gear train, which must adhere to a set of mechanical constraints to ensure functionality, safety, and manufacturability.

Mathematical formulation The weight of the speed reducer is expressed as the objective function to be minimized, as shown in Eq. 30, which includes face width, gear module, number of teeth, and the diameters of the shafts.

$$\begin{aligned} f(\mathbf{x}) = & 0.7854 \times x_1 \times (x_2^2) \times (3.3333 \times x_3^2 + 14.9334 \times x_3 - 43.0934) \\ & - 1.508 \times x_1 \times (x_6^2 + x_7^2) + 7.4777 \times (x_6^3 + x_7^3) \\ & + 0.7854 \times (x_4 \times (x_6^2) + x_5 \times (x_7^2)). \end{aligned} \tag{30}$$

The speed reducer is controlled by many constraints, as shown in Eq. 37-47, whereas these constraints represent the physical and performance-related requirements such as stress limits, deflection limits, gear ratios, and dimensions corresponding to the practical engineering limits. Furthermore, a penalty function is incorporated into the optimization routine to impose a significant cost on designs that do not meet the constraints.

$$\frac{27}{x_1 \times (x_2^2) \times x_3} \leq 1, \tag{31}$$

$$\frac{397.5}{x_1 \times (x_2^2) \times (x_3^2)} \leq 1, \tag{32}$$

$$\frac{1.93 \times x_4^3}{x_2 \times x_3 \times (x_6^4)} \leq 1, \tag{33}$$

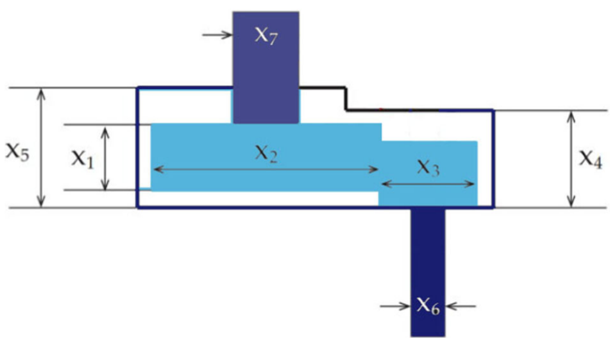

Fig. 20 Speed reducer design

$$\frac{1.93 \times x_5^3}{x_2 \times x_3 \times (x_7^4)} \leq 1, \tag{34}$$

$$\frac{745 \times x_4}{x_2 \times x_3} \times \left(\frac{1}{110 \times x_6^3}\right)^{0.5} \leq 1, \tag{35}$$

$$\frac{745 \times x_5}{x_2 \times x_3} \times \left(\frac{1}{85 \times x_7^3}\right)^{0.5} \leq 1, \tag{36}$$

$$\frac{x_2 \times x_3}{40} \leq 1, \tag{37}$$

$$\frac{5 \times x_2}{x_1} \leq 1, \tag{38}$$

$$\frac{x_1}{12 \times x_2} \leq 1, \tag{39}$$

$$\frac{1.5 \times x_6 + 1.9}{x_4} \leq 1, \tag{40}$$

$$\frac{1.1 \times x_7 + 1.9}{x_5} \leq 1. \tag{41}$$

The results in Table 16 indicate that FDA achieves the best performance, reaching the global optimum solution of 2994.47 with the lowest standard deviation of 4.12×10^{-12} for solving the speed reducer design problem. WSO and EAO closely follow FDA, with mean values of 2994.48 and 2994.49, respectively, and small standard deviations of 3.03×10^{-2} for WSO and 7.60×10^{-3} for EAO, demonstrating strong reliability and stability in convergence. Moreover, GTO and DE also perform competitively, reaching near-optimal solutions but with slightly larger standard deviations, ranking 4th and 5th, respectively. However, optimizers such as GMO, MFO, and GWO display higher mean values and variability, indicating less consistent results. WOA, HLOA, and HGSO exhibit significantly larger deviations and mean values, indicating their lower stability in this optimization problem.

7.4 Cantilever beam design

The cantilever beam design problem (see Fig. 21) is a structural engineering optimization problem that aims to minimize the weight of a beam while ensuring it can bear certain loading conditions. The design of the beam is defined by the dimensions of its cross-sectional areas, which are critical in determining the beam's deflection under load.

Mathematical formulation The weight of the cantilever beam is to be minimized as shown in Eq. 42:

Table 16 Optimization results of the speed reducer design with optimal variable values (X1-X7)

Optimizer	Mean	Std	Max	Best score	X1	X2	X3	X4	X5	X6	X7	Rank
EAO	2994.49	7.60E-03	2994.52	2994.48	3.5000	0.7000	17.0000	7.3000	7.7156	3.3502	5.2867	3
WOA	3234.77	4.00E+02	5492.57	3010.97	3.5013	0.7000	17.0000	7.7531	7.9285	3.3782	5.2867	11
MFO	3006.33	1.75E+01	3043.08	2994.47	3.5000	0.7000	17.0000	7.3000	7.7153	3.3502	5.2867	8
MVO	3035.28	1.78E+01	3085.39	3004.45	3.5030	0.7001	17.0000	7.3013	8.0349	3.3520	5.2879	9
SCA	3129.69	4.06E+01	3200.73	3046.10	3.5129	0.7000	17.0000	7.3834	8.2530	3.4138	5.3139	10
GWO	3006.16	4.04E+00	3016.38	2998.20	3.5005	0.7000	17.0059	7.3272	7.7823	3.3526	5.2869	7
WSO	2994.48	3.03E-02	2994.67	2994.47	3.5000	0.7000	17.0000	7.3000	7.7153	3.3502	5.2867	2
HGSO	3421.54	7.92E+02	5843.30	3112.42	3.5021	0.7001	17.0007	7.8236	7.9545	3.3781	5.2867	13
SHO	3126.83	3.44E+01	3192.03	3064.39	3.6000	0.7000	17.0000	7.3000	8.3000	3.3770	5.3038	11
HLOA	3356.42	8.24E+02	5722.30	2994.65	3.5001	0.7000	17.0007	7.3013	7.7161	3.3502	5.2867	12
FDA	2994.47	4.12E-12	2994.47	2994.47	3.5000	0.7000	17.0000	7.3000	7.7153	3.3502	5.2867	1
GTO	2996.66	5.71E+00	3016.77	2994.47	3.5000	0.7000	17.0000	7.3000	7.7153	3.3502	5.2867	4
GMO	2999.14	4.45E+00	3012.66	2994.49	3.5000	0.7000	17.0000	7.3000	7.7164	3.3502	5.2867	6
PSO	3077.73	6.70E+01	3222.26	2994.47	3.5000	0.7000	17.0000	7.3000	7.7153	3.3502	5.2867	10
DE	2997.55	1.02E+01	3033.75	2994.47	3.5000	0.7000	17.0000	7.3000	7.7153	3.3502	5.2867	5

$$f(\mathbf{x}) = 0.0624 \times (x_1 + x_2 + x_3 + x_4 + x_5) \tag{42}$$

where x_1, x_2, x_3, x_4, and x_5 represent the beam's sectional dimensions.

Moreover, the cantilever beam design is constrained by the allowable deflection as shown in Eq. 43, which ensures that the deflection of the beam under the applied load does not exceed a specified limit.

$$\frac{61}{x_1^3} + \frac{37}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} \leq 1 \tag{43}$$

Furthermore, a penalty is applied to the objective function to handle any violation of this constraint, as shown in Eq. 44:

$$\text{Penalty} = 10^5 \times (\text{sum of violations}) \tag{44}$$

where the sum of violations is the aggregated measure of the extent to which the constraints are exceeded.

This penalty term is essential, as it guides the optimization algorithm to search for solutions within the feasible design space, thus ensuring that the resultant beam design is not only light but also structurally sound.

The results in Table 17 demonstrate that EAO and WSO achieve the best performance, both obtaining the global optimum solution of 1.339956 and sharing the top rank for solving the cantilever design problem. WSO exhibits the lowest standard deviation of 5.695×10^{-11}, followed by EAO with 7.204×10^{-9}, indicating exceptional stability in convergence. DE also performs well, achieving the same best solution with a slightly higher standard deviation of 1.496×10^{-5},

Fig. 21 Cantilever beam design

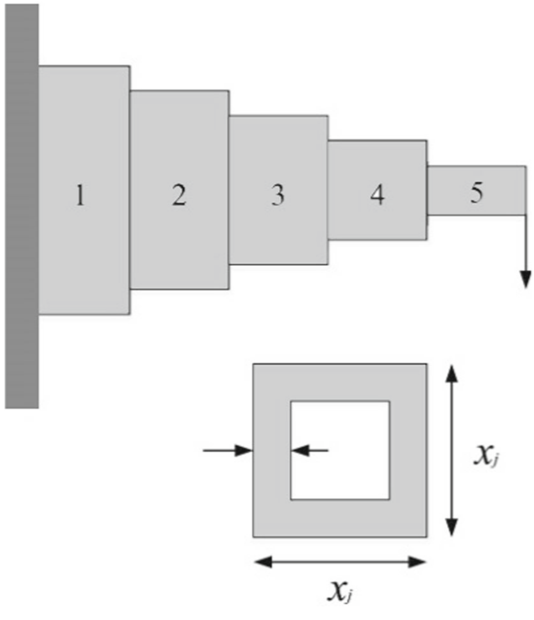

Table 17 Optimization results of the cantilever design with optimal variable values (X1-X5)

Optimizer	Mean	Std	Max	Best	X1	X2	X3	X4	X5	Rank
EAO	1.339956	7.204E-09	1.339956	1.339956	6.015861	5.308478	4.494518	3.502251	2.152552	1
WOA	1.421810	6.492E-02	1.650366	1.346084	5.749414	5.575874	4.314355	3.594273	2.337943	14
MFO	1.340975	8.721E-04	1.344858	1.340130	5.978099	5.307265	4.524242	3.473866	2.192970	10
MVO	1.340024	5.111E-05	1.340211	1.339967	6.017746	5.301604	4.498034	3.499232	2.157213	6
SCA	1.378782	1.657E-02	1.424709	1.350794	5.982733	5.408117	4.123456	3.659398	2.473636	12
GWO	1.340032	6.304E-05	1.340309	1.339964	6.011165	5.315015	4.497800	3.496157	2.153637	7
WSO	1.339956	5.695E-11	1.339956	1.339956	6.016016	5.309174	4.494330	3.501475	2.152665	1
HGSO	1.462610	8.262E-02	1.680163	1.348246	5.846422	5.553295	4.436125	3.523152	2.369125	15
SHO	1.359791	7.069E-03	1.371370	1.344490	5.966440	5.218305	4.848131	3.467963	2.045482	11
HLOA	1.386573	7.595E-02	1.789162	1.340358	6.101689	5.211225	4.489877	3.506664	2.170637	13
FDA	1.340079	1.786E-04	1.341069	1.339958	6.014600	5.307749	4.496366	3.498637	2.156329	9
GTO	1.339997	6.917E-05	1.340315	1.339957	6.018976	5.308420	4.494343	3.499156	2.152771	5
GMO	1.340046	7.553E-05	1.340241	1.339961	6.025274	5.313531	4.488185	3.496975	2.149767	8
PSO	1.339991	3.443E-05	1.340110	1.339959	6.009442	5.311429	4.497776	3.498387	2.156670	4
DE	1.339962	1.496E-05	1.340022	1.339956	6.016016	5.309174	4.494330	3.501475	2.152665	3

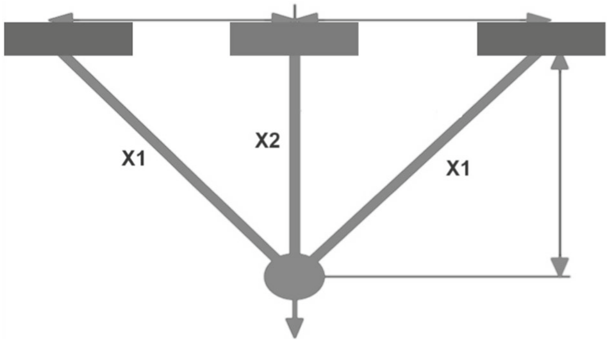

Fig. 22 Three bars design

placing it in the third rank. PSO and GTO also closely approximate the optimal solution, with marginally larger standard deviations. On the other hand, MVO, GWO, and GMO display slightly higher mean values and variability, indicating less consistent performance. Optimizers such as WOA, SCA, and HGSO exhibit significantly larger deviations and higher mean values.

7.5 Three bars design

The three bars design problem, as shown in Fig. 22, is an optimization problem in structural engineering focused on minimizing the weight of a truss structure while maintaining structural integrity under specified loading conditions. This problem is essential for optimizing material usage and ensuring stability in lightweight structures.

Mathematical formulation The objective function, representing the total weight of the three bars, is to be minimized, as shown in Eq. 45:

$$f(\mathbf{x}) = (2 \times \sqrt{2} \times x_1 + x_2) \times 100 \tag{45}$$

where x_1 and x_2 are the cross-sectional areas of the bars. x_1 typically represents the area of two bars at an angle (assumed for simplification to be symmetrically placed), and x_2 represents the area of a horizontal bar connecting these two.

The design is constrained by three conditions that relate to the stress and displacement limits of the structure, as shown in Eq. 52–54:

$$\frac{\sqrt{2} \times x_1 + x_2}{\sqrt{2} \times x_1^2 + 2 \times x_1 \times x_2} \times 2 - 2 \leq 0, \tag{46}$$

$$\frac{x_2}{\sqrt{2} \times x_1^2 + 2 \times x_1 \times x_2} \times 2 - 2 \leq 0, \tag{47}$$

Table 18 Optimization results of the three bars design with optimal variable values (X1-X2)

Optimizer	Mean	Std	Max	Best	X1	X2	Rank
EAO	263.8958	2.87E-13	263.8958	263.8958	0.788675	0.408248	1
WOA	264.5491	8.27E-01	268.3066	263.8975	0.787183	0.412485	12
MFO	263.9670	1.59E-01	264.9597	263.8960	0.788250	0.409451	10
MVO	263.8987	2.74E-03	263.9070	263.8960	0.788948	0.407478	7
SCA	265.1560	4.51E+00	282.8427	263.8979	0.787523	0.411527	14
GWO	263.8990	2.84E-03	263.9088	263.8959	0.788776	0.407963	8
WSO	263.8958	2.54E-07	263.8958	263.8958	0.788675	0.408248	3
HGSO	265.2140	3.22E+00	282.8536	263.8977	0.786821	0.416226	15
SHO	264.7501	3.73E+00	282.8427	263.9062	0.785004	0.418737	13
HLOA	264.0218	4.63E-01	266.2466	263.8958	0.788615	0.408419	11
FDA	263.8960	8.15E-04	263.9016	263.8958	0.788676	0.408245	5
GTO	263.8958	1.86E-05	263.8958	263.8958	0.788747	0.408045	4
GMO	263.9097	2.47E-02	264.0365	263.8958	0.788595	0.408475	9
PSO	263.8970	1.554E-03	263.9018	263.8958	0.788704	0.408167	6
DE	263.8958	2.586E-12	263.8958	263.8958	0.788675	0.408248	1

$$\frac{1}{x_1 + \sqrt{2} \times x_2} \times 2 - 2 \leq 0. \tag{48}$$

The results in Table 18 demonstrate that EAO, DE, WSO and GTO achieve the best performance for the three bars design problem, each convergent to the global optimum solution of 263.8958, where EAO exhibits the lowest standard deviation of 2.87×10^{-13}, making it the most stable optimizer. Moreover, DE follows with a slightly higher standard deviation of 2.586×10^{-12}, while WSO (2.54×10^{-7}) and GTO (1.86×10^{-5}) also show strong stability.

On the other hand, PSO achieves a slightly higher mean value of 263.8970 with a standard deviation of 1.554×10^{-3}, indicating minor variations in convergence. Other optimizers such as GMO, MFO, and HLOA display higher mean values and standard deviations, indicating less consistent performance. Optimizers like WOA, SCA, and HGSO show significantly larger deviations.

8 Conclusions and future directions

This paper proposed a novel bio-inspired optimization algorithm called enzyme action optimization (EAO), motivated by the adaptive mechanisms of enzymatic actions in biological systems. By simulating enzymatic behaviors, EAO maintains a dynamic balance between exploration and exploitation, as simulated to the way enzymes in biological systems adapt their catalytic rates and substrate values. EAO performance has been evaluated across a diverse benchmark functions, including the IEEE CEC (2017, and 2022) and the classical 23 functions benchmark functions, experimental results demonstrate EAO's superiority in achieving rapid convergence and high-quality solutions, even in complex and high-dimensional fitness landscapes. Moreover, when applied to real-world engineering tasks such as pressure vessel design, tension/compression spring design, speed reducer design, cantilever beam design, and three-bar truss design, EAO exhibits strong performance compared to state-of-the-art optimization algorithms.

The design of EAO draws on empirical insights that highlight its ability to maintain a robust balance between exploration and exploitation throughout the optimization process. Its dual focus on adaptivity and diversity preservation is reflected in two key mechanisms. First, the Adaptive Factor dynamically controls the exploration-exploitation trade-off, allowing EAO to explore deeply into promising areas without becoming trapped in local optima. Second, the enzyme-substrate interaction paradigm ensures that both promising and less-explored regions of the search space are carefully sampled. This combination of targeted search and global coverage explains EAO's strong performance on a wide range of problems and shows its versatility as a general-purpose optimizer.

EAO's capacity to be deployed with minimal parameter tuning is particularly appealing for real-world applications, where limited a priori knowledge or evolving problem dynamics often render extensive manual tuning impractical. EAO's adaptability thus allows it to remain efficient and robust when problem constraints or landscapes shift, a frequent occurrence in dynamic engineering or industrial contexts.

Future developments of EAO will concentrate on further expanding its applicability and enhancing its capabilities. A binary version of EAO could solve problems such as feature selection, thereby extending EAO's utility to domains where the design space is discrete and the decision variables are inherently binary. An additional extension lies in the field of multi-objective optimization, where EAO would manage simultaneous trade-offs among multiple objectives, thus allowing broader applicability to complex real-world challenges that require the optimization of conflicting objectives. Investigations into hybridization with other metaheuristics or machine learning models, including deep neural networks, could also unlock new strategies for adaptive parameter control and enhanced diversity management. Moreover, exploring parallelized or distributed implementations of EAO may offer substantial gains in computational efficiency and scalability for large-scale, high-dimensional problems. Finally, applying EAO to dynamic optimization tasks where the objective function or constraints shift over time presents a significant opportunity to refine EAO's real-time adaptability. Such directions promise to both enrich EAO's theoretical foundations and expand the scope of its practical applications.

Appendix A Wilcoxon signed-rank test significant results over the studies benchmark datasets

See Tables 19, 20, 21.

Table 19 Wilcoxon signed-rank test results over 23 classical functions

Function	Index	WOA	MVO	SCA	GWO	WSO	HGSO	FDA	GMO	GTO	HLOA	SHO	MFO	DE	PSO
F1	p-value	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	
	T+/T-	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F2	p-value	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	
	T+/T-	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F3	p-value	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	
	T+/T-	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F4	p-value	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	9.84E-03	
	T+/T-	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	358/107	
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F5	p-value	1.73E-06	2.60E-06	2.84E-05	1.73E-06	2.13E-06	8.47E-06	1.73E-06	1.73E-06	2.35E-06	1.73E-06	5.22E-06	2.13E-06	6.73E-01	1.73E-06
	T+/T-	0/465	461/4	436/29	465/0	2/463	16/449	465/0	465/0	3/462	465/0	454/11	2/463	253/212	465/0
	Winner	-	+	+	+	-	-	+	+	-	+	+	-	=	+
F6	p-value	1.36E-05	3.82E-01	1.73E-06	1.92E-06	8.31E-04	1.73E-06	1.73E-06	1.92E-06	1.73E-06	1.73E-06	3.39E-01	8.31E-04	3.11E-05	1.73E-06
	T+/T-	21/444	275/190	465/0	464/1	395/70	465/0	465/0	464/1	465/0	465/0	279/186	395/70	435/30	465/0
	Winner	-	=	+	+	+	+	+	+	+	+	=	+	+	+

Table 19 continued

Function	Index	WOA	MVO	SCA	GWO	WSO	HGSO	FDA	GMO	GTO	HLOA	SHO	MFO	DE	PSO
F7	p-value	1.02E-05	1.73E-06	1.73E-06	1.73E-06	4.07E-02	1.73E-06	1.73E-06	1.73E-06	8.13E-01	1.73E-06	1.73E-06	4.07E-02	1.73E-06	7.71E-04
	T+/T-	447/18	465/0	465/0	465/0	332/133	465/0	465/0	465/0	244/221	465/0	465/0	465/0	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	=	+	+	+	+	+
F8	p-value	1.73E-06	1.73E-06	2.35E-06	6.34E-06	9.26E-01	6.56E-02	1.73E-06	1.73E-06	6.42E-03	1.92E-06	1.73E-06	9.26E-01	5.22E-06	4.86E-05
	T+/T-	0/465	0/465	462/3	13/452	237/228	322/143	0/465	0/465	365/100	464/1	0/465	237/228	454/11	430/35
	Winner	-	-	+	-	=	=	-	-	+	+	-	=	+	+
F9	p-value	1.00E+00	1.73E-06	1.73E-06	1.73E-06	1.56E-02	1.73E-06	1.73E-06	1.73E-06	1.00E+00	1.73E-06	1.73E-06	1.56E-02	1.73E-06	5.75E-06
	T+/T-	0/0	465/0	465/0	465/0	17/0	465/0	465/0	465/0	0/0	465/0	465/0	17/0	465/0	453/12
	Winner	=	+	+	+	+	+	+	+	=	+	+	+	+	+
F10	p-value	3.56E-05	1.73E-06	1.73E-06	1.73E-06	1.73E-06	2.57E-07	1.73E-06	1.73E-06	2.57E-07	1.73E-06	1.73E-06	1.73E-06	2.85E-02	1.57E-02
	T+/T-	45/0	465/0	465/0	465/0	465/0	35/0	465/0	465/0	35/0	465/0	465/0	465/0	126/339	115/350
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	-	-
F11	p-value	3.13E-02	1.73E-06	1.73E-06	1.73E-06	5.00E-01	2.44E-04	1.73E-06	1.73E-06	1.00E+00	1.73E-06	1.73E-06	5.00E-01	7.50E-01	1.02E-01
	T+/T-	27/0	465/0	465/0	465/0	5/0	104/0	465/0	465/0	0/0	465/0	465/0	5/0	217/248	312/153
	Winner	+	+	+	+	=	+	+	+	=	+	+	=	=	=

Table 19 continued

Function	Index	WOA	MVO	SCA	GWO	WSO	HGSO	FDA	GMO	GTO	HLOA	SHO	MFO	DE	PSO
F12	p-value	1.73E-06	3.11E-05	1.73E-06	1.73E-06	4.07E-02	4.90E-04	1.73E-06	1.73E-06	2.13E-06	1.73E-06	5.75E-06	4.07E-02	2.37E-01	1.73E-06
	T+/T-	0/465	435/30	465/0	465/0	332/133	402/63	465/0	465/0	463/2	465/0	453/12	332/133	175/290	0/465
	Winner	-	+	+	+	+	+	+	+	+	+	+	+	=	-
F13	p-value	6.34E-06	1.73E-06	1.73E-06	1.73E-06	4.53E-04	1.73E-06	9.32E-06	7.51E-05	1.73E-06	1.73E-06	1.92E-06	4.53E-04	2.60E-06	4.73E-06
	T+/T-	13/452	0/465	465/0	465/0	403/62	465/0	17/448	40/425	465/0	465/0	1/464	403/62	461/4	455/10
	Winner	-	-	+	+	+	+	-	-	+	+	-	+	+	+
F14	p-value	1.92E-06	1.73E-06	1.73E-06	2.61E-04	2.13E-06	1.73E-06	3.18E-01	8.61E-01	1.73E-06	1.73E-06	3.88E-06	2.13E-06	5.75E-06	1.92E-06
	T+/T-	464/1	465/0	465/0	410/55	463/2	465/0	281/184	241/224	465/0	465/0	457/8	463/2	453/12	464/1
	Winner	+	+	+	+	+	+	=	=	+	+	+	+	+	+
F15	p-value	6.34E-06	2.88E-06	1.73E-06	1.02E-05	9.37E-02	3.18E-01	1.85E-02	7.86E-02	5.71E-04	2.22E-04	1.36E-05	9.37E-02	1.48E-04	3.60E-01
	T+/T-	452/13	460/5	465/0	447/18	151/314	281/184	347/118	318/147	65/400	412/53	444/21	151/314	417/48	277/188
	Winner	+	+	+	+	=	=	+	=	-	+	+	=	+	=
F16	p-value	1.73E-06	6.64E-04	1.92E-06	1.73E-06	1.73E-06	3.59E-04	5.75E-06	4.29E-06	2.41E-04	2.05E-04	8.94E-04	1.73E-06	3.00E-02	1.20E-03
	T+/T-	0/465	67/398	464/1	0/465	0/465	59/406	12/453	9/456	54/411	413/52	71/394	0/465	338/127	390/75
	Winner	-	-	+	-	-	-	-	-	-	+	-	-	+	+

Table 19 continued

Function	Index	WOA	MVO	SCA	GWO	WSO	HGSO	FDA	GMO	GTO	HLOA	SHO	MFO	DE	PSO
F17	p-value	3.52E-06	3.18E-06	1.73E-06	1.73E-06	9.92E-01	6.89E-05	1.73E-06	3.52E-06	3.39E-01	2.35E-06	2.13E-06	9.92E-01	1.73E-06	5.79E-05
	T+/T-	7/458	6/459	465/0	0/465	233/232	39/426	0/465	7/458	186/279	462/3	2/463	233/232	465/0	428/37
	Winner	-	-	+	-	=	-	-	-	=	+	-	=	+	+
F18	p-value	2.45E-01	1.11E-03	1.59E-01	1.73E-06	1.92E-06	3.00E-02	2.60E-06	3.88E-04	6.98E-06	8.22E-03	7.51E-05	1.92E-06	1.73E-06	1.38E-03
	T+/T-	176/289	74/391	301/164	0/465	1/464	338/127	4/461	60/405	14/451	104/361	40/425	1/464	465/0	388/77
	Winner	=	-	=	-	-	+	-	-	-	-	-	-	+	+
F19	p-value	2.60E-06	1.73E-06	1.73E-06	1.73E-06	1.32E-02	2.13E-01	1.73E-06	1.92E-06	5.79E-05	1.73E-06	1.92E-06	1.32E-02	2.88E-06	4.20E-04
	T+/T-	461/4	0/465	465/0	0/465	353/112	293/172	0/465	1/464	428/37	465/0	1/464	353/112	460/5	404/61
	Winner	+	-	+	-	+	=	-	-	+	+	-	+	+	+
F20	p-value	4.65E-01	2.89E-01	1.73E-06	5.45E-02	7.71E-04	1.71E-01	2.13E-01	2.18E-02	1.36E-05	1.73E-06	1.02E-01	7.71E-04	1.73E-06	7.51E-05
	T+/T-	268/197	284/181	465/0	139/326	396/69	299/166	172/293	121/344	444/21	465/0	312/153	396/69	465/0	425/40
	Winner	=	=	+	=	+	=	=	-	+	+	=	+	+	+
F21	p-value	6.32E-05	1.36E-04	1.73E-06	2.60E-05	1.24E-05	1.15E-04	4.07E-05	2.11E-03	2.84E-05	1.73E-06	1.97E-05	1.24E-05	1.73E-06	1.73E-06
	T+/T-	427/38	418/47	465/0	437/28	445/20	420/45	432/33	382/83	436/29	465/0	440/25	445/20	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 19 continued

Function	Index	WOA	MVO	SCA	GWO	WSO	HGSO	FDA	GMO	GTO	HLOA	SHO	MFO	DE	PSO
F22	p-value	1.92E-06	1.97E-05	1.73E-06	1.02E-01	1.73E-06	2.35E-06	4.20E-04	4.86E-05	1.73E-06	1.73E-06	9.71E-05	1.73E-06	2.13E-06	1.48E-04
	T+/T-	464/1	440/25	465/0	312/153	465/0	462/3	404/61	430/35	465/0	465/0	422/43	465/0	463/2	417/48
	Winner	+	+	+	=	+	+	+	+	+	+	+	+	+	+
F23	p-value	2.88E-06	9.71E-05	1.73E-06	8.73E-03	1.73E-06	1.73E-06	6.42E-03	3.16E-02	1.73E-06	1.73E-06	3.88E-06	1.73E-06	1.73E-06	1.00E+00
	T+/T-	460/5	422/43	465/0	360/105	465/0	465/0	365/100	337/128	465/0	465/0	457/8	465/0	465/0	0/0
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	=
Total	**[±/=]**	13/7/3	15/6/2	22/0/1	16/5/2	16/3/4	16/3/4	15/6/2	14/7/2	15/4/4	22/1/0	15/6/2	16/3/4	19/1/3	18/2/3

Table 20 Wilcoxon signed-rank test results over CEC2017 functions

Function	Index	WOA	MFO	MVO	SCA	GWO	WSO	HGSO	SHO	HLOA	FDA	GTO	GMO	PSO	DE
F1	p-value	1.73E-06	2.35E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	4.07E-05	1.36E-01	1.89E-04	1.32E-02	1.73E-06	7.69E-06
	T+/T-	465/0	462/3	465/0	465/0	465/0	465/0	465/0	465/0	432/33	305/160	414/51	353/112	465/0	450/15
	Winner	+	+	+	+	+	+	+	+	+	=	+	+	+	+
F2	p-value	1.73E-06	2.13E-06	6.34E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.71E-03	1.73E-06	1.73E-06	1.92E-06	1.73E-06	1.73E-06
	T+/T-	465/0	463/2	452/13	465/0	465/0	0/465	465/0	465/0	385/80	0/465	0/465	464/1	465/0	465/0
	Winner	+	+	+	+	+	-	+	+	+	-	-	+	+	+
F3	p-value	1.73E-06	8.22E-03	8.61E-01	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	2.43E-02	1.73E-06	1.73E-06	1.73E-06	1.73E-06	2.41E-03
	T+/T-	465/0	361/104	241/224	465/0	465/0	465/0	465/0	465/0	123/342	0/465	0/465	0/465	465/0	85/380
	Winner	+	+	=	+	+	+	+	+	-	-	-	-	+	-
F4	p-value	1.73E-06	2.13E-06	1.13E-05	1.73E-06	1.73E-06	8.31E-04	1.73E-06	2.13E-06	6.34E-06	3.11E-05	2.60E-06	1.73E-06	1.32E-02	2.07E-02
	T+/T-	465/0	463/2	446/19	465/0	465/0	395/70	465/0	463/2	452/13	30/435	4/461	465/0	353/112	345/120
	Winner	+	+	+	+	+	+	+	+	+	-	-	+	+	+
F5	p-value	1.92E-06	1.73E-06	1.83E-03	1.73E-06	7.50E-01	6.58E-01	1.73E-06	2.60E-06	1.92E-06	3.11E-05	2.22E-04	4.95E-02	2.13E-06	1.92E-06
	T+/T-	464/1	465/0	384/81	465/0	248/217	254/211	465/0	461/4	464/1	435/30	412/53	328/137	2/463	1/464
	Winner	+	+	+	+	=	=	+	+	+	+	+	+	-	-
F6	p-value	1.73E-06	6.64E-04	4.29E-06	1.73E-06	5.31E-05	6.98E-06	1.73E-06	1.73E-06	1.73E-06	2.35E-06	1.73E-06	3.60E-01	3.18E-06	1.02E-05
	T+/T-	465/0	398/67	456/9	465/0	429/36	14/451	465/0	465/0	465/0	3/462	465/0	277/188	459/6	447/18

Table 20 continued

Function	Index	WOA	MFO	MVO	SCA	GWO	WSO	HGSO	SHO	HLOA	FDA	GTO	GMO	PSO	DE
	Winner	+	+	+	+	+	-	+	+	+	-	+	=	+	+
F7	p-value	1.73E-06	3.18E-06	9.37E-02	1.73E-06	4.07E-02	8.47E-06	1.73E-06	1.73E-06	1.73E-06	7.69E-06	5.75E-06	6.73E-01	6.87E-02	2.45E-01
	T+/T-	465/0	459/6	314/151	465/0	332/133	16/449	465/0	465/0	465/0	450/15	453/12	253/212	144/321	176/289
	Winner	+	+	=	+	+	-	+	+	+	+	+	=	=	=
F8	p-value	2.60E-06	8.47E-06	4.86E-05	1.73E-06	1.31E-01	6.14E-01	1.73E-06	2.88E-06	1.73E-06	6.89E-05	3.52E-06	2.29E-01	7.51E-05	1.06E-04
	T+/T-	461/4	449/16	430/35	465/0	306/159	257/208	465/0	460/5	465/0	426/39	458/7	291/174	425/40	421/44
	Winner	+	+	+	+	=	=	+	+	+	+	+	=	+	+
F9	p-value	1.73E-06	1.36E-05	6.88E-01	1.73E-06	1.36E-05	3.68E-02	1.73E-06	1.73E-06	1.73E-06	2.29E-01	2.60E-06	2.84E-05	2.60E-05	1.64E-05
	T+/T-	465/0	444/21	213/252	465/0	444/21	131/334	465/0	465/0	465/0	291/174	461/4	29/436	437/28	442/23
	Winner	+	+	=	+	+	-	+	+	+	=	+	-	+	+
F10	p-value	1.48E-02	4.28E-01	1.11E-03	3.41E-05	7.27E-03	4.53E-04	1.73E-06	4.53E-01	3.88E-06	7.19E-02	3.60E-01	2.62E-01	5.45E-02	1.15E-04
	T+/T-	351/114	271/194	74/391	434/31	102/363	62/403	465/0	196/269	457/8	145/320	277/188	178/287	139/326	45/420
	Winner	+	=	-	+	-	-	+	=	+	=	=	=	=	-
F11	p-value	1.73E-06	1.92E-06	1.73E-06	1.73E-06	1.73E-06	6.34E-06	1.73E-06	1.73E-06	1.73E-06	1.06E-04	1.73E-06	1.73E-06	7.50E-01	7.19E-02
	T+/T-	465/0	464/1	465/0	465/0	465/0	13/452	465/0	465/0	465/0	421/44	465/0	465/0	217/248	145/320
	Winner	+	+	+	+	+	-	+	+	+	+	+	+	=	=
F12	p-value	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	3.38E-03	1.73E-06	1.73E-06	1.73E-06	2.88E-06	1.92E-06	1.73E-06	1.73E-06	1.73E-06

Table 20 continued

Function	Index	WOA	MFO	MVO	SCA	GWO	WSO	HGSO	SHO	HLOA	FDA	GTO	GMO	PSO	DE
	T+/T-	465/0	465/0	465/0	465/0	465/0	90/375	465/0	465/0	465/0	460/5	464/1	465/0	0/465	0/465
	Winner	+	+	+	+	+	-	+	+	+	+	+	+	-	-
F13	p-value	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.25E-04	1.73E-06	1.73E-06	1.73E-06	1.06E-04	6.89E-05	1.73E-06	6.98E-06	2.85E-02
	T+/T-	465/0	465/0	465/0	465/0	465/0	46/419	465/0	465/0	465/0	421/44	426/39	465/0	451/14	339/126
	Winner	+	+	+	+	+	-	+	+	+	+	+	+	+	+
F14	p-value	1.73E-06	1.92E-06	5.29E-04	1.73E-06	1.73E-06	1.48E-02	1.73E-06	1.73E-06	1.73E-06	2.13E-06	1.49E-05	1.73E-06	2.60E-06	3.18E-06
	T+/T-	465/0	464/1	401/64	465/0	465/0	351/114	465/0	465/0	465/0	463/2	443/22	465/0	461/4	459/6
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F15	p-value	1.73E-06	1.73E-06	2.16E-05	1.73E-06	1.73E-06	6.89E-05	1.73E-06	1.73E-06	1.92E-06	4.45E-05	1.80E-05	1.73E-06	1.57E-02	1.31E-01
	T+/T-	465/0	465/0	439/26	465/0	465/0	39/426	465/0	465/0	464/1	431/34	441/24	465/0	350/115	306/159
	Winner	+	+	+	+	+	-	+	+	+	+	+	+	+	=
F16	p-value	1.73E-06	2.60E-06	1.13E-05	1.73E-06	3.52E-06	5.71E-04	1.73E-06	2.88E-06	1.92E-06	9.27E-03	8.22E-03	1.73E-06	1.97E-05	1.31E-01
	T+/T-	465/0	461/4	446/19	465/0	458/7	65/400	465/0	460/5	464/1	359/106	361/104	465/0	440/25	306/159
	Winner	+	+	+	+	+	-	+	+	+	+	+	+	+	=
F17	p-value	2.13E-06	1.85E-02	2.37E-01	1.73E-06	3.00E-02	1.73E-06	1.73E-06	2.70E-02	1.73E-06	4.20E-04	3.39E-01	2.13E-06	9.32E-06	1.80E-05
	T+/T-	463/2	347/118	290/175	465/0	338/127	465/0	465/0	340/125	465/0	61/404	279/186	463/2	448/17	441/24
	Winner	+	+	=	+	+	+	+	+	+	-	=	+	+	+
F18	p-value	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	2.35E-06	1.02E-05	1.73E-06	4.53E-04	1.25E-02

Table 20 continued

Function	Index	WOA	MFO	MVO	SCA	GWO	WSO	HGSO	SHO	HLOA	FDA	GTO	GMO	PSO	DE
	T+/T-	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	462/3	447/18	465/0	403/62	354/111
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F19	p-value	1.73E-06	1.73E-06	2.16E-05	1.73E-06	1.73E-06	6.32E-05	1.73E-06	1.92E-06	1.73E-06	1.36E-05	1.13E-05	1.73E-06	1.64E-05	2.05E-04
	T+/T-	465/0	465/0	439/26	465/0	465/0	427/38	465/0	464/1	465/0	444/21	446/19	465/0	442/23	413/52
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F20	p-value	1.73E-06	9.78E-02	6.64E-04	1.73E-06	1.04E-03	1.73E-06	1.73E-06	1.80E-05	1.73E-06	1.60E-04	2.58E-03	2.35E-06	6.32E-05	2.26E-03
	T+/T-	465/0	313/152	398/67	465/0	392/73	465/0	465/0	441/24	465/0	49/416	379/86	462/3	427/38	381/84
	Winner	+	=	+	+	+	+	+	+	+	-	+	+	+	+
F21	p-value	1.64E-05	5.31E-05	1.60E-04	3.88E-04	4.45E-05	1.73E-06	3.16E-02	1.48E-04	1.49E-05	3.33E-02	8.77E-01	1.48E-04	2.60E-05	3.52E-06
	T+/T-	442/23	429/36	416/49	405/60	431/34	465/0	337/128	417/48	443/22	336/129	240/225	417/48	437/28	458/7
	Winner	+	+	+	+	+	+	+	+	+	+	=	+	+	+
F22	p-value	1.73E-06	2.58E-03	4.07E-05	5.22E-06	6.98E-06	1.73E-06	1.73E-06	1.73E-06	3.52E-06	2.41E-04	2.18E-02	2.56E-02	5.67E-03	1.71E-01
	T+/T-	465/0	379/86	432/33	454/11	451/14	465/0	465/0	465/0	458/7	411/54	344/121	341/124	367/98	166/299
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	=
F23	p-value	1.73E-06	4.73E-06	7.19E-02	1.73E-06	9.84E-03	1.73E-06	1.73E-06	1.73E-06	1.73E-06	3.06E-04	8.92E-05	5.72E-01	1.00E+00	1.00E+00
	T+/T-	465/0	455/10	320/145	465/0	358/107	465/0	465/0	465/0	465/0	408/57	423/42	205/260	0/0	0/0
	Winner	+	+	=	+	+	+	+	+	+	+	+	=	=	=

Table 20 continued

Function	Index	WOA	MFO	MVO	SCA	GWO	WSO	HGSO	SHO	HLOA	FDA	GTO	GMO	PSO	DE
F24	p-value	1.89E-04	1.59E-03	2.41E-04	1.73E-06	5.29E-04	7.19E-01	2.60E-05	5.67E-03	2.16E-05	3.72E-05	3.71E-01	1.25E-01	1.73E-06	1.92E-06
	T+/T-	414/51	386/79	411/54	465/0	401/64	250/215	28/437	367/98	439/26	433/32	276/189	158/307	465/0	464/1
	Winner	+	+	+	+	+	=	-	+	+	+	=	=	+	+
F25	p-value	3.11E-05	3.88E-06	4.07E-05	1.73E-06	1.92E-06	1.73E-06	1.73E-06	1.73E-06	2.13E-06	1.24E-05	2.35E-06	2.13E-06	1.73E-06	1.73E-06
	T+/T-	435/30	457/8	432/33	465/0	464/1	465/0	465/0	465/0	463/2	445/20	462/3	463/2	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F26	p-value	5.79E-05	4.73E-06	2.37E-05	1.73E-06	1.13E-05	1.73E-06	1.73E-06	2.35E-06	7.69E-06	4.72E-02	4.07E-02	1.73E-06	6.32E-05	5.72E-01
	T+/T-	428/37	455/10	438/27	465/0	446/19	0/465	465/0	462/3	450/15	329/136	332/133	0/465	427/38	260/205
	Winner	+	+	+	+	+	-	+	+	+	+	+	-	+	=
F27	p-value	1.73E-06	1.73E-06	4.07E-05	1.73E-06	1.92E-06	2.35E-06	1.73E-06	1.73E-06	1.73E-06	4.73E-06	1.92E-06	1.73E-06	1.73E-06	1.92E-06
	T+/T-	465/0	465/0	432/33	465/0	464/1	462/3	465/0	465/0	465/0	455/10	464/1	465/0	465/0	464/1
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F28	p-value	1.73E-06	1.92E-06	5.75E-06	1.73E-06	1.73E-06	7.69E-06	1.73E-06	5.22E-06	7.69E-06	2.61E-04	2.61E-04	6.16E-04	1.00E+00	1.00E+00
	T+/T-	465/0	464/1	453/12	465/0	465/0	450/15	465/0	454/11	450/15	410/55	410/55	399/66	0/0	0/0
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	=	=
F29	p-value	1.73E-06	1.97E-05	8.47E-06	1.73E-06	3.41E-05	1.24E-05	1.73E-06	1.73E-06	1.73E-06	6.32E-05	1.04E-03	1.02E-05	1.73E-06	3.18E-06
	T+/T-	465/0	440/25	449/16	465/0	434/31	445/20	465/0	465/0	465/0	427/38	392/73	447/18	465/0	459/6
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 20 continued

Function	Index	WOA	MFO	MVO	SCA	GWO	WSO	HGSO	SHO	HLOA	FDA	GTO	GMO	PSO	DE
F30	p-value	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	2.35E-06	1.73E-06	1.73E-06	3.41E-05	4.73E-06	1.73E-06	1.73E-06	1.92E-06
	T+/T-	465/0	465/0	465/0	465/0	465/0	465/0	462/3	465/0	465/0	434/31	455/10	465/0	465/0	464/1
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+
Total	**[±/=]**	30/0/0	28/0/2	24/1/5	30/0/0	27/1/2	16/11/3	29/1/0	29/0/1	29/1/0	21/6/3	23/3/4	21/3/6	23/2/5	18/4/8

Table 21 Wilcoxon signed-rank test results over CEC2022 benchmark functions

Function	Index	WOA	MFO	MVO	SCA	GWO	WSO	HGSO	SHO	HLOA	FDA	GTO	GMO	DE	PSO
F1	p-value	1.73E-06	2.06E-01	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	8.22E-02	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	T+/T-	465/0	294/171	465/0	465/0	465/0	465/0	465/0	465/0	317/148	0/465	0/465	0/465	465/0	465/0
	Winner	+	=	+	+	+	+	+	+	=	-	-	-	+	+
F2	p-value	2.37E-05	4.86E-05	2.37E-05	1.73E-06	1.80E-05	1.29E-03	1.73E-06	8.94E-04	6.04E-03	3.60E-01	2.43E-02	7.51E-05	1.73E-06	1.73E-06
	T+/T-	438/27	430/35	438/27	465/0	441/24	76/389	465/0	394/71	366/99	188/277	342/123	40/425	465/0	465/0
	Winner	+	+	+	+	+	-	+	+	+	=	+	-	+	+
F3	p-value	1.73E-06	8.73E-03	2.13E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	2.83E-04	1.73E-06	6.56E-02	1.73E-06	1.73E-06
	T+/T-	465/0	360/105	463/2	465/0	465/0	465/0	465/0	465/0	465/0	56/409	465/0	322/143	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	+	-	+	=	+	+
F4	p-value	1.73E-06	5.22E-06	1.38E-03	1.73E-06	2.99E-01	1.49E-05	1.73E-06	9.32E-06	1.73E-06	2.37E-05	4.29E-06	7.04E-01	1.73E-06	1.73E-06
	T+/T-	465/0	454/11	388/77	465/0	283/182	22/443	465/0	448/17	465/0	438/27	456/9	214/251	465/0	465/0
	Winner	+	+	+	+	=	-	+	+	+	+	+	=	+	+
F5	p-value	1.73E-06	1.36E-05	1.36E-01	1.73E-06	1.92E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	3.18E-06	1.73E-06	1.73E-06	4.73E-06	1.73E-06
	T+/T-	465/0	444/21	160/305	465/0	464/1	465/0	465/0	465/0	465/0	459/6	465/0	0/465	10/455	465/0
	Winner	+	+	=	+	+	+	+	+	+	+	+	-	-	+
F6	p-value	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.80E-05	1.73E-06	1.73E-06	1.73E-06	5.22E-06	2.88E-06	1.73E-06	3.52E-06	1.73E-06

Table 21 continued

Function	Index	WOA	MFO	MVO	SCA	GWO	WSO	HGSO	SHO	HLOA	FDA	GTO	GMO	DE	PSO
	T+/T-	465/0	465/0	465/0	465/0	465/0	24/441	465/0	465/0	465/0	454/11	460/5	465/0	7/458	465/0
	Winner	+	+	+	+	+	-	+	+	+	+	+	+	-	+
F7	p-value	1.73E-06	4.95E-02	3.49E-01	1.73E-06	1.25E-02	1.11E-03	1.73E-06	6.64E-04	1.73E-06	3.82E-01	2.85E-02	2.18E-02	1.36E-05	1.73E-06
	T+/T-	465/0	328/137	278/187	465/0	354/111	74/391	465/0	398/67	465/0	190/275	339/126	344/121	444/21	465/0
	Winner	+	+	=	+	+	-	+	+	+	=	+	+	+	+
F8	p-value	1.73E-06	2.84E-05	7.16E-04	1.73E-06	8.92E-05	6.42E-03	1.73E-06	9.32E-06	1.73E-06	3.59E-04	4.86E-05	2.60E-06	1.73E-06	1.73E-06
	T+/T-	465/0	436/29	397/68	465/0	423/42	365/100	465/0	448/17	465/0	406/59	430/35	461/4	0/465	465/0
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	-	+
F9	p-value	1.73E-06	6.73E-01	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	9.75E-01	1.73E-06	3.11E-05	1.73E-06	2.06E-01	1.73E-06
	T+/T-	465/0	212/253	465/0	465/0	465/0	465/0	465/0	465/0	234/231	0/465	30/435	465/0	171/294	465/0
	Winner	+	=	+	+	+	+	+	+	=	-	-	+	=	+
F10	p-value	1.73E-06	1.73E-06	3.59E-04	1.73E-06	5.31E-05	1.59E-03	1.73E-06	1.73E-06	1.73E-06	4.73E-06	5.75E-06	1.25E-04	1.79E-06	1.73E-06
	T+/T-	465/0	465/0	406/59	465/0	429/36	386/79	465/0	465/0	465/0	455/10	453/12	419/46	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F11	p-value	1.73E-06	1.36E-05	1.73E-06	1.73E-06	1.73E-06	6.73E-01	1.73E-06	1.73E-06	1.15E-04	9.75E-01	3.71E-01	3.71E-01	2.45E-01	1.73E-06
	T+/T-	465/0	444/21	465/0	465/0	465/0	212/253	465/0	465/0	420/45	234/231	189/276	189/276	176/289	465/0
	Winner	+	+	+	+	+	=	+	+	+	=	=	=	=	+

Table 21 continued

Function	Index	WOA	MFO	MVO	SCA	GWO	WSO	HGSO	SHO	HLOA	FDA	GTO	GMO	DE	PSO
F12	p-value	1.73E-06	5.22E-06	3.50E-02	1.73E-06	4.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.06E-04	5.75E-06	1.73E-06	1.73E-06	1.73E-06
	T+/T-	465/0	454/11	335/130	465/0	455/10	465/0	465/0	465/0	465/0	421/44	453/12	465/0	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+
Total	**[±/=]**	12/0/0	10/0/2	10/0/2	12/0/0	11/0/1	7/4/1	12/0/0	12/0/0	10/0/2	6/3/3	9/2/1	6/3/3	7/3/2	12/0/0

Appendix B The 23 classical benchmark functions)

See Tables 22, 23, 24.

Table 22 Unimodal functions (F1-F7) of the 23 classical benchmark functions

Name	Equation	Dim	Range	f_{opt}
F_1	$\sum_{i=1}^{n} x_i^2$	30	$[-100, 100]$	0
F_2	$\sum_{i=1}^{n} \lvert x_i\rvert + \prod_{i=1}^{n} \lvert x_i\rvert$	30	$[-10, 10]$	0
F_3	$\sum_{i=1}^{n} \left(\sum_{j=1}^{i} x_j\right)^2$	30	$[-100, 100]$	0
F_4	$\max_{1 \le i \le n} \lvert x_i\rvert$	30	$[-100, 100]$	0
F_5	$\sum_{i=1}^{n-1} \left(100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2\right)$	30	$[-30, 30]$	0
F_6	$\sum_{i=1}^{n} (x_i + 0.5)^2$	30	$[-100, 100]$	0
F_7	$\sum_{i=1}^{n} ix_i^4 + \text{random}[0, 1)$	30	$[-1.28, 1.28]$	0

Table 23 Multimodal functions (F8-F13) of the 23 classical benchmark functions

Name	Equation	Dim	Range	f_{opt}
F_8	$\sum_{i=1}^{n} -x_i \sin(\sqrt{\lvert x_i \rvert})$	30	[− 500,500]	−418.9829 × 5
F_9	$\sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i) + 10]$	30	[−5.12,5.12]	0
F_{10}	$-20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$	30	[− 32,32]	0
F_{11}	$\frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	30	[− 600,600]	0
F_{12}	$\frac{\pi}{n}\left(10\sin^2(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10\sin^2(\pi y_{i+1})] + (y_n - 1)^2\right) + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$	30	[− 50,50]	0
F_{13}	$0.1\left[\sin^2(3\pi N_1) + \sum_{i=1}^{n-1}(x_i - 1)^2\left(1 + \sin^2(3\pi x_{i+1})\right) + (x_n - 1)^2\left(1 + \sin^2(2\pi x_n)\right)\right] + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$	30	[− 50,50]	0

Table 24 Fixed-dimension multimodal functions (F14-F23) of the 23 classical benchmark functions

Name	Equation	Dim	Range	f_{opt}
F_{14}	$\left(\frac{1}{500}+\sum_{j=1}^{25}\left(\frac{1}{j+\sum_{i=1}^{2}(x_i-a_{ij})^6}\right)\right)^{-1}$	2	[-65,65]	1
F_{15}	$\sum_{i=1}^{11}\left\|a_i-\frac{N_1(b_i^2+b_ix_2)}{b_i^2+b_ix_3+x_4}\right\|^2$ 0.00030	4	[-5,5]	
F_{16}	$4N_1^2-2.1N_1^4+\frac{1}{3}N_1^6+N_1x_2-4x_2^2+4x_2^4$ -1.0316	2	[-5,5]	
F_{17}	$(x_2-\frac{5.1}{4\pi^2}N_1^2+\frac{5}{\pi}N_1-6)^2+10(1-\frac{1}{8\pi})\cos N_1+10$	2	[-5,5]	
0.398				
F_{18}	$(1+(N_1+N_2+1)^2(19-14N_1+3N_1^2-$ $14x_2+6N_1x_2+3x_2^2))\times$ $(30+(2N_1-3x_2)^2(18-32N_1+12N_1^2+48x_2-36N_1x_2+27x_2^2))$	2	[-2,2]	3
F_{19}	$-\sum_{i=1}^{4}c_i\exp(-\sum_{j=1}^{3}a_{ij}(x_j-p_{ij})^2)$ -3.86	3	[1,3]	
F_{20}	$-\sum_{i=1}^{4}c_i\exp(-\sum_{j=1}^{6}a_{ij}(x_j-p_{ij})^2)$ -3.32	6	[0,1]	
F_{21}	$-\sum_{i=1}^{5}[(X-a_i)(X-a_i)^T+c_i^{-1}]$ -10.1532	4	[0,10]	
F_{22}	$-\sum_{i=1}^{7}[(X-a_i)(X-a_i)^T+c_i^{-1}]$ -10.4028	4	[0,10]	
F_{23}	$-\sum_{i=1}^{10}[(X-a_i)(X-a_i)^T+c_i^{-1}]$ -10.5363	4	[0,10]	

Appendix C IEEE CEC benchmark functions

CEC 2017 functions

See Table 25.

Table 25 IEEE CEC2017 Benchmarks

	F No	Functions	f_{opt}
Unimodal	1	Shifted and rotated Bent Cigar function	100
functions	2	Shifted and rotated Zakharov function	200
	3	Shifted and rotated Rosenbrock's function	300
	4	Shifted and rotated Rastrigin's function	400
	5	Shifted and rotated expanded Scaffer's F6 function	500
Simple multimodal	6	Shifted and rotated Lunacek Bi_Rastrigin function	600
functions	7	Shifted and rotated Non-Continuous Rastrigin's function	700
	8	Shifted and rotated Levy function	800
	9	Shifted and rotated Schwefel's function	900
	10	Hybrid function 1 (N - 3)	1000
	11	Hybrid function 2 (N = 3)	1100
	12	Hybrid function 3 (N = 3)	1200
	13	Hybrid function 4 (N = 4)	1300
Hybrid	14	Hybrid function 5 (N = 4)	1400
functions	15	Hybrid function 6 (N = 5)	1500
	16	Hybrid function 6 (N = 5)	1600
	17	Hybrid function 6 (N = 5)	1700
	18	Hybrid function 6 (N = 6)	1800
	19	Hybrid function 6 (N = 6)	1900
	20	Composition function 1 (N = 3)	2000
	21	Composition function 2 (N = 3)	2100
	22	Composition function 3 (N = 4)	2200
	23	Composition function 4 (N = 4)	2300
Composition	24	Composition function 5 (N = 5)	2400
functions	25	Composition function 6 (N = 5)	2500
	26	Composition function 7 (N = 6)	2600
	27	Composition function 8 (N = 6)	2700
	28	Composition function 9 (N = 3)	2800
	29	Composition function 10 (N = 3)	2900

CEC 2022 functions

See Table 26.

Table 26 IEEE CEC2022 Benchmark functions

Type	No	Function Name	f_{opt}
Unimodal function	F1	Shifted and full Rotated Zakharov function	300
	F2	Shifted and full Rotated Rosenbrock's function	400
Multimodal functions	F3	Shifted and full Rotated Expanded Schaffer's f6 function	600
	F4	Shifted and full Rotated Non-Continuous Rastrigin's function	800
	F5	Shifted and full Rotated Levy function	900
Hybrid functions	F6	Hybrid function 1 ($N = 3$)	1800
	F7	Hybrid function 2 ($N = 6$)	2000
	F8	Hybrid function 3 ($N = 5$)	2200
Composition functions	F9	Composition function 1 ($N = 5$)	2300
	F10	Composition function 2 ($N = 4$)	2400
	F11	Composition function 3 ($N = 6$)	2600
	F12	Composition function 4 ($N = 6$)	2700

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Data availability Enzyme action optimization (EAO) code is available for both MATLAB and PYTHON at: EAO MATLAB code Link and EAO PYTHON code Link.

Declarations

Conflict of interest The authors declare no conflict of interest.

References

1. Sharma M, Kaur P (2021) A comprehensive analysis of nature-inspired meta-heuristic techniques for feature selection problem. Arch Comput Method Eng 28:1103–1127
2. Wang Z, Schafer BC (2020) Machine leaming to set meta-heuristic specific parameters for high-level synthesis design space exploration, In: 57th ACM/IEEE Design Automation Conference (DAC). IEEE, pp 1–6
3. Dokeroglu T, Sevinc E, Kucukyilmaz T, Cosar A (2019) A survey on new generation metaheuristic algorithms. Comput Ind Eng 137:106040
4. Vikhar PA (2016) Evolutionary algorithms: A critical review and its future prospects. In: International Conference on Global Trends in Signal Processing, Information Computing and Communication (ICGTSPICC). IEEE, pp 261–265

5. Storn R, Price K (1997) Differential evolution - a simple and efficient heuristic for global optimization over continuous spaces. J Glob Optim 11:341–359
6. Beni G (2020) Swarm intelligence. In: Sotomayor M, Pérez-Castrillo D, Castiglione F (eds) Complex social and behavioral systems: game theory and agent-based models. Springer, New York, NY, pp 791–818
7. Wang D, Tan D, Liu L (2018) Particle swarm optimization algorithm: an overview. Soft Comput 22:387–408
8. Dorigo M, Stützle T (2010) Ant colony optimization: overview and recent advances. In: Gendreau M, Potvin J-Y (eds) Handbook of metaheuristics. Springer, Boston, MA, pp 227–263
9. Rashedi E, Rashedi E, Nezamabadi-Pour H (2018) A comprehensive survey on gravitational search algorithm. Swarm Evolutionary Comput 41:141–158
10. Mirjalili S, Jangir P, Saremi S (2017) Multi-objective ant lion optimizer: a multi-objective optimization algorithm for solving engineering problems. Appl Intell 46:79–95
11. Suresh S, Lal S (2017) Multilevel thresholding based on chaotic Darwinian particle swarm optimization for segmentation of satellite images. Appl Soft Comput 55:503–522
12. Kumar A, Pant S, Ram M, Yadav O (2022) Meta-heuristic optimization techniques: applications in engineering, Vol 10, Walter de Gruyter GmbH & Co KG
13. Mirjalili S, Mirjalili SM, Lewis A (2014) Grey wolf optimizer. Adv Eng Softw 69:46–61
14. Brest J, Maučec MS, Bošković B, Single objective real-parameter optimization: Algorithm jso, In: (2017) IEEE Congress on Evolutionary Computation (CEC). IEEE 2017:1311–1318
15. Copeland RA (2023) Enzymes: a practical introduction to structure, mechanism, and data analysis. John Wiley & Sons, Hoboken
16. Punekar N (2018) Enzymes: catalysis, kinetics and mechanisms. Springer Nature, Singapore
17. Mirjalili S, Jangir P, Mirjalili SZ, Saremi S, Trivedi IN (2017) Optimization of problems with multiple objectives using the multi-verse optimization algorithm. Knowl Based Syst 134:50–71
18. Kennedy J, Eberhart R Particle swarm optimization, In: Proceedings of ICNN'95-International Conference on Neural Networks, vol 4, ieee, 1995, pp 1942–1948
19. Dorigo M, Birattari M, Stutzle T (2006) Ant colony optimization. IEEE Comput Intell Mag 1:28–39
20. Karaboga D, Basturk B (2005) A powerful and efficient algorithm for numerical function optimization: artificial bee colony (abc) algorithm. J Glob Optim 39(3):459–471
21. Mirjalili S (2015) The ant lion optimizer. Adv Eng Softw 83:80–98
22. Chou J-S, Truong D-N (2021) A novel metaheuristic optimizer inspired by behavior of jellyfish in ocean. Appl Math Comput 389:125535
23. Akbari MA, Zare M, Azizipanah-Abarghooee R, Mirjalili S, Deriche M (2022) The cheetah optimizer: a nature-inspired metaheuristic algorithm for large-scale optimization problems. Sci Rep 12:10953
24. Xian S, Feng X (2023) Meerkat optimization algorithm: a new meta-heuristic optimization algorithm for solving constrained engineering problems. Expert Syst Appl 231:120482
25. Heidari AA et al (2019) Harris hawks optimization: algorithm and applications. Futur Gener Comput Syst 97:849–872
26. Kirkpatrick S, Gelatt CD, Vecchi MP (1983) Optimization by simulated annealing. Science 220:671–680
27. Rashedi E, Nezamabadi-Pour H, Saryazdi S (2009) GSA: a gravitational search algorithm. Information Sci 179(13):2232–2248
28. Mirjalili S, Mirjalili SM, Hatamlou A (2016) Multi-verse optimizer: a nature-inspired algorithm for global optimization. Neural Comput Appl 27:495–513
29. Hatamlou A (2013) Black hole: a new heuristic optimization approach for data clustering. Inf Sci (N Y) 222:175–184
30. Zhao W, Wang L, Zhang Z (2019) Atom search optimization and its application to solve a hydrogeologic parameter estimation problem. Knowl Based Syst 163:283–304
31. Lam AYS, Li VOK (2009) Chemical-reaction-inspired metaheuristic for optimization. IEEE Trans Evol Comput 14:381–399
32. Wei Z, Huang C, Wang X, Han T, Li Y (2019) Nuclear reaction optimization: a novel and powerful physics-based algorithm for global optimization. IEEE Access 7:66084–66109
33. Holland JH (1975) Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control, and Artificial Intelligence, University of Michigan Press, Ann Arbor, MI, introduced the concept of Genetic Algorithms (GA)

34. Banzhaf W, Francone FD, Keller RE, Nordin P (1998) Genetic programming: an introduction: on the automatic evolution of computer programs and its applications, Morgan Kaufmann Publishers Inc
35. Abualigah L, Diabat A, Mirjalili S, Abd Elaziz M, Gandomi AH (2021) The arithmetic optimization algorithm. Comput Methods Appl Mech Eng 376:113609
36. Talatahari S, Azizi M (2021) Chaos game optimization: a novel metaheuristic algorithm. Artif Intell Rev 54:917–1004
37. Mirjalili S (2016) SCA: a sine cosine algorithm for solving optimization problems. Knowl Based Syst 96:120–133
38. Deng L, Liu S (2023) Snow ablation optimizer: a novel metaheuristic technique for numerical optimization and engineering design. Expert Syst Appl 225:120069
39. Eskandar H, Sadollah A, Bahreininejad A, Hamdi M (2012) Water cycle algorithm - a novel metaheuristic optimization method for solving constrained engineering optimization problems. Comput Struct 110–111:151–166
40. Trojovská E, Dehghani M (2022) A new human-based metahurestic optimization method based on mimicking cooking training. Sci Rep 12:14861
41. Reynolds R (1994) An introduction to cultural algorithms
42. Dehghani M, Montazeri Z, Givi H, Guerrero J, Dhiman G (2020) Darts game optimizer: a new optimization technique based on darts game. Int J Intell Eng Syst 13:286–294
43. Zeidabadi FA, Dehghani M (2022) POA: Puzzle Optimization Algorithm. Int J Intell Eng Syst 15(1)
44. Azizi M, Baghalzadeh Shishehgarkhaneh M, Basiri M, Moehler RC (2023) Squid game optimizer (SGO): a novel metaheuristic algorithm. Sci Rep 13:5373
45. Zhao W, Wang L, Zhang Z (2019) Supply-demand-based optimization: a novel economics-inspired algorithm for global optimization. IEEE Access 7:73182–73206
46. Shabani A, Asgarian B, Salido M, Asil Gharebaghi S (2020) Search and rescue optimization algorithm: a new optimization method for solving constrained engineering optimization problems. Expert Syst Appl 161:113698
47. Slowik A, Kwasnicka H (2020) Evolutionary algorithms and their applications to engineering problems. Neural Comput Appl 32:12363–12379
48. Li G, Zhang T, Tsai C-Y, Yao L, Lu Y, Tang J (2024) Review of the metaheuristic algorithms in applications: visual analysis based on bibliometrics. Expert Syst Applicat 255:124857
49. Mishra A, Goel L (2024) Metaheuristic algorithms in smart farming: an analytical survey. IETE Tech Rev 41(1):46–65
50. Sharma P, Raju S (2024) Metaheuristic optimization algorithms: a comprehensive overview and classification of benchmark test functions. Soft Comput 28(4):3123–3186
51. Tomar V, Bansal M, Singh P (2024) Metaheuristic algorithms for optimization: a brief review. Eng Proc 59(1):238
52. Mirjalili S, Lewis A (2016) The whale optimization algorithm. Adv Eng Softw 95:51–67
53. Pan Q, Tang J, Zhan J, Li H (2023) Bacteria phototaxis optimizer. Neural Comput Appl 35(18):13433–13464
54. Pan Q, Tang J, Lao S (2022) Edoa: an elastic deformation optimization algorithm. Appl Intell 52(15):17580–17599
55. Pan Q, Wang H, Tang J, Lv Z, Wang Z, Wu X, Ruan Y, Yv T, Lao M (2024) Eioa: a computing expectation-based influence evaluation method in weighted hypergraphs. Inf Process Manag 61(6):103856
56. Leveson-Gower RB, Mayer C, Roelfes G (2019) The importance of catalytic promiscuity for enzyme design and evolution. Nat Rev Chem 3(12):687–705
57. Sauro HM (2011) Enzyme kinetics for systems biology, Future Skill Software
58. Peraza-Vázquez H, Peña-Delgado A, Merino-Treviño M, Morales-Cepeda AB, Sinha N (2024) A novel metaheuristic inspired by horned lizard defense tactics. Artif Intell Rev 57(3):59
59. Zhao S, Zhang T, Ma S, Wang M (2023) Sea-horse optimizer: a novel nature-inspired meta-heuristic for global optimization problems. Appl Intell 53(10):11833–11860
60. Rezaei F, Safavi HR, Abd Elaziz M, Mirjalili S (2023) GMO: geometric mean optimizer for solving engineering problems. Soft Comput 27(15):10571–10606
61. Braik M, Hammouri A, Atwan J, Al-Betar MA, Awadallah MA (2022) White shark optimizer: a novel bio-inspired meta-heuristic algorithm for global optimization problems. Knowl Based Syst 243:108457

62. Abdollahzadeh B, Soleimanian Gharehchopogh F, Mirjalili S (2021) Artificial gorilla troops optimizer: a new nature-inspired metaheuristic algorithm for global optimization problems. Int J Intell Syst 36(10):5887–5958
63. Karami H, Anaraki MV, Farzin S, Mirjalili S (2021) Flow direction algorithm (FDA): a novel optimization approach for solving optimization problems. Comput Ind Eng 156:107224
64. Hashim FA, Houssein EH, Mabrouk MS, Al-Atabany W, Mirjalili S (2019) Henry gas solubility optimization: a novel physics-based algorithm. Future Gener Comput Syst 101:646–667
65. Mirjalili S (2015) Moth-flame optimization algorithm: a novel nature-inspired heuristic paradigm. Knowl Based Syst 89:228–249

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